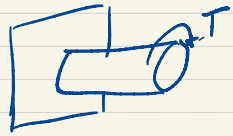
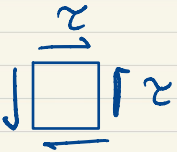
بسم الله الرحمن الرحیم

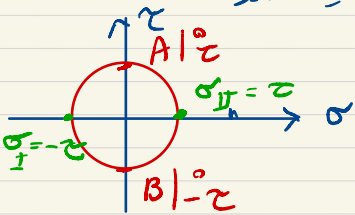
مراحل یک

مسئله ۹

آزمایش پیچش ساده:







$\sigma_I = -\tau \;,\; \sigma_{II} = \tau \;,\; \sigma_{III} = 0$

$$\frac{4\tau^2 + \tau^2 + \tau^2}{2} \leq S_y^2 \longrightarrow \tau = \frac{S_y}{\sqrt{3}} \Rightarrow \tau_{max} = 0.577\, S_y$$

ضریب اطمینان:

$$n = \frac{\text{استحکام}}{\text{تنش موجود}}$$

۱- معیار تنش نرمال ماکزیمم

$$n = \frac{S_y}{\sigma_{III}} \leftarrow \text{تنش اصلی ماکزیمم}$$

۲- معیار ترسکا (ماکزیمم تنش برشی)

$$n = \frac{S_y/2}{\tau_{max}} \longrightarrow \tau_{max} = \frac{S_y/2}{n}$$

۳- معیار فون مایز

تنش معادل فون مایز

$$\bar{\sigma} = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}{2}}$$

$$n = \frac{S_y}{\bar{\sigma}} \longrightarrow \bar{\sigma} = \frac{S_y}{n}$$

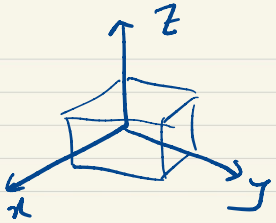
$$\bar{\sigma} = \sqrt{I_1^2 - 3I_2}$$

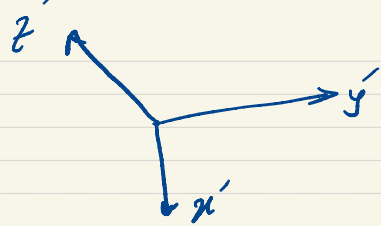
$$I_3 = det(\sigma)$$

$$I_1 = \sigma_x + \sigma_y + \sigma_z$$

$$I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{xz}^2 - \tau_{yz}^2$$

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ & \sigma_{yy} & \sigma_{yz} \\ sym & & \sigma_{zz} \end{bmatrix}$$





$$\sigma' = \begin{bmatrix} \sigma_{x'x'} & \sigma_{x'y'} & \sigma_{x'z'} \\ & \sigma_{y'y'} & \sigma_{y'z'} \\ \text{Sym} & & \sigma_{z'z'} \end{bmatrix}$$

$$I_1 = I_1'$$

$$I_2 = I_2'$$

$$I_3 = I_3'$$

$$\bar{\sigma} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2}$$

$$\bar{\sigma} = \sqrt{\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2}$$

در حالت دوبعدی:

معیارهای شکست به زبان نمودار:

برای مواد نرم (ductile)

1- maximum shear stress theory (ترسکا)

2- maximum distortion energy theory (فون مایزز)

برای مواد ترد (brittle)

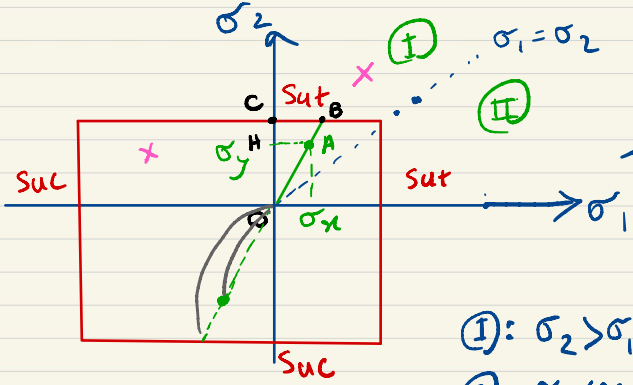
1- maximum normal stress theory

2- internal friction theory

3- modified mohr theory

① ماکزیم تنش نرمال (brittle 1)

مواد ترد در فشار و کشش رفتار یکسانی از خود نشان نمی دهند (در فشار تحمل بیشتری دارند)



استحکام نهایی ماده در کشش : S_{ut}

استحکام نهایی ماده در فشار : S_{uc}

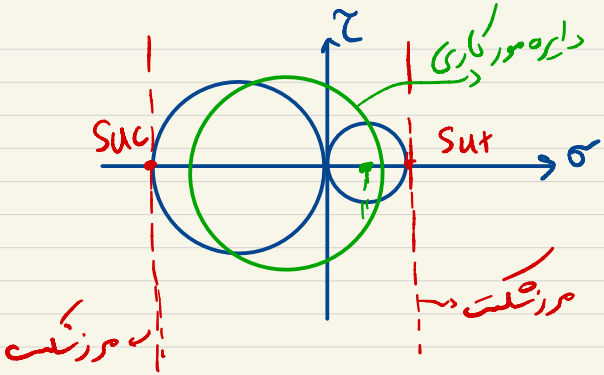
$A \left| \begin{matrix} \sigma_x \\ \sigma_y \end{matrix} \right.$

تنش ماکزیم

I: $\sigma_2 > \sigma_1 \longrightarrow \sigma_2 < S_{ut}$

II: $\sigma_2 < \sigma_1 \longrightarrow \sigma_1 < S_{ut}$

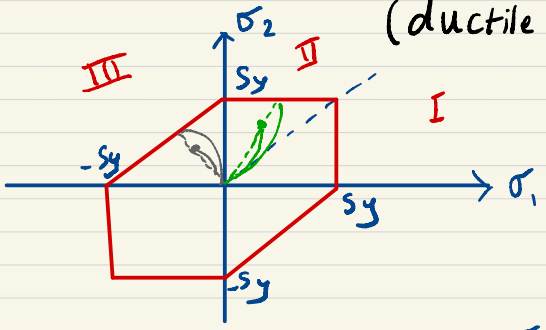
$$n = \frac{S_{ut}}{\sigma_y} = \frac{OC}{OH} = \frac{OB}{OA}$$



در دستگاه مختصات مور (Mohr):

هر دایره موری که در این ناحیه بیافتد تسلیم نمی‌شود.

② - ماکزیمم تنش برشی (ترسکا) (ductile 1)

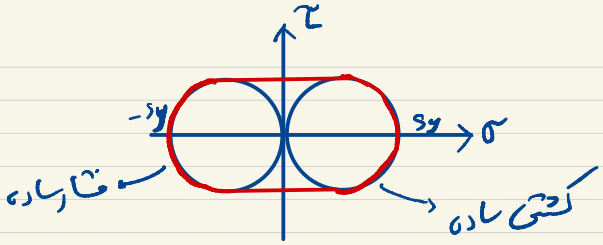


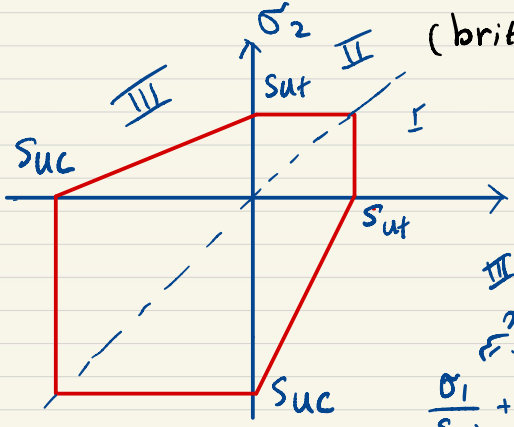
$$\tau_{max} = \frac{\text{تنش اصلی ماکزیمم} - \text{تنش اصلی مینیمم}}{2}$$

(I) $\sigma_1 > \sigma_2 > 0$ $\quad \tau_{max} = \frac{\sigma_1 - 0}{2} = \frac{S_y}{2} \rightarrow \sigma_1 = S_y$

(II) $\sigma_2 > \sigma_1 > 0$ $\quad \tau_{max} = \frac{\sigma_2 - 0}{2} = \frac{S_y}{2} \rightarrow \sigma_2 = S_y$

(III) $\sigma_2 > 0 > \sigma_1$ $\quad \tau_{max} = \frac{\sigma_2 - \sigma_1}{2} = \frac{S_y}{2} \rightarrow \sigma_2 - \sigma_1 = S_y$





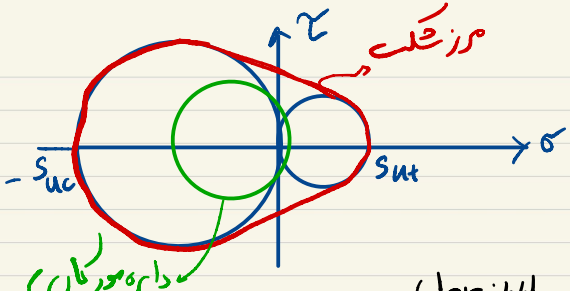
(brittle 2) internal friction theory ③
(Coulomb-Mohr)
همان معیار ترسکا برای مواد ترد است.

I), II) $\sigma_{max} \lesssim S_{ut}$

III) $\frac{\sigma_1}{S_{ut}} + \frac{\sigma_2}{S_{uc}} = 1$

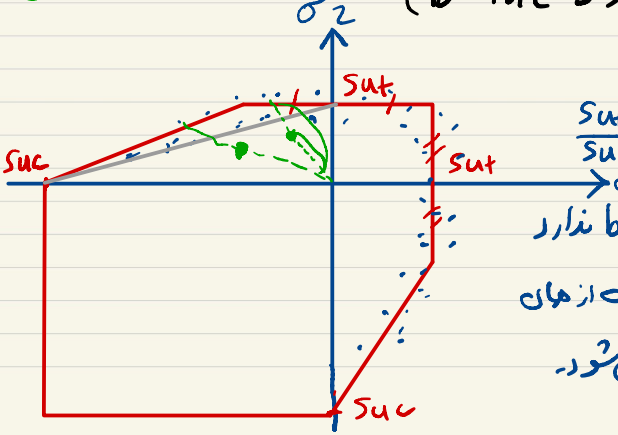
اگر ضریب اطمینان n را وارد کنیم

$$\frac{\sigma_1}{S_{ut}} + \frac{\sigma_2}{S_{uc}} = \frac{1}{n}$$



: mohr

④ modified mohr theary (brittle 3)



این معیار بصورت تجربی بدست آمده است با توجه به اینکه در مواد ترد $\frac{S_{ut}}{S_{uc}} \approx \frac{1}{5}$

این تئوری تفاوت زیادی با تئوری brittle 2 ندارد

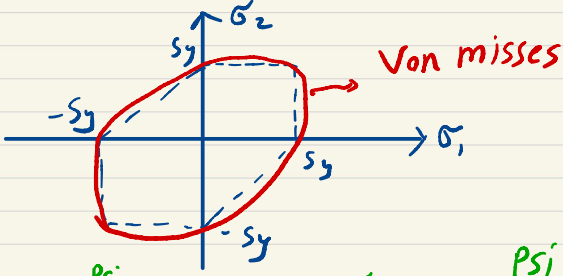
لذا برای راحتی محاسبات از همان brittle 2 استفاده می‌شود.

⑤ فون مایز (Von misses):

اگر $\sigma_3 = 0$ باشد آنگاه رابطه فون میز به صورت زیر در می آید:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = S_y^2$$

که معادله یک بیضی است:



تمرین: در یک ماده نرم که $S_y = 4000^{psi}$ و با بارگذاری $\sigma_x = 1300^{psi}$ و $\sigma_y = 300^{psi}$ و $\tau_{xy} = 1200^{psi}$ ضریب اطمینان را از دو تئوری بدست آورید.

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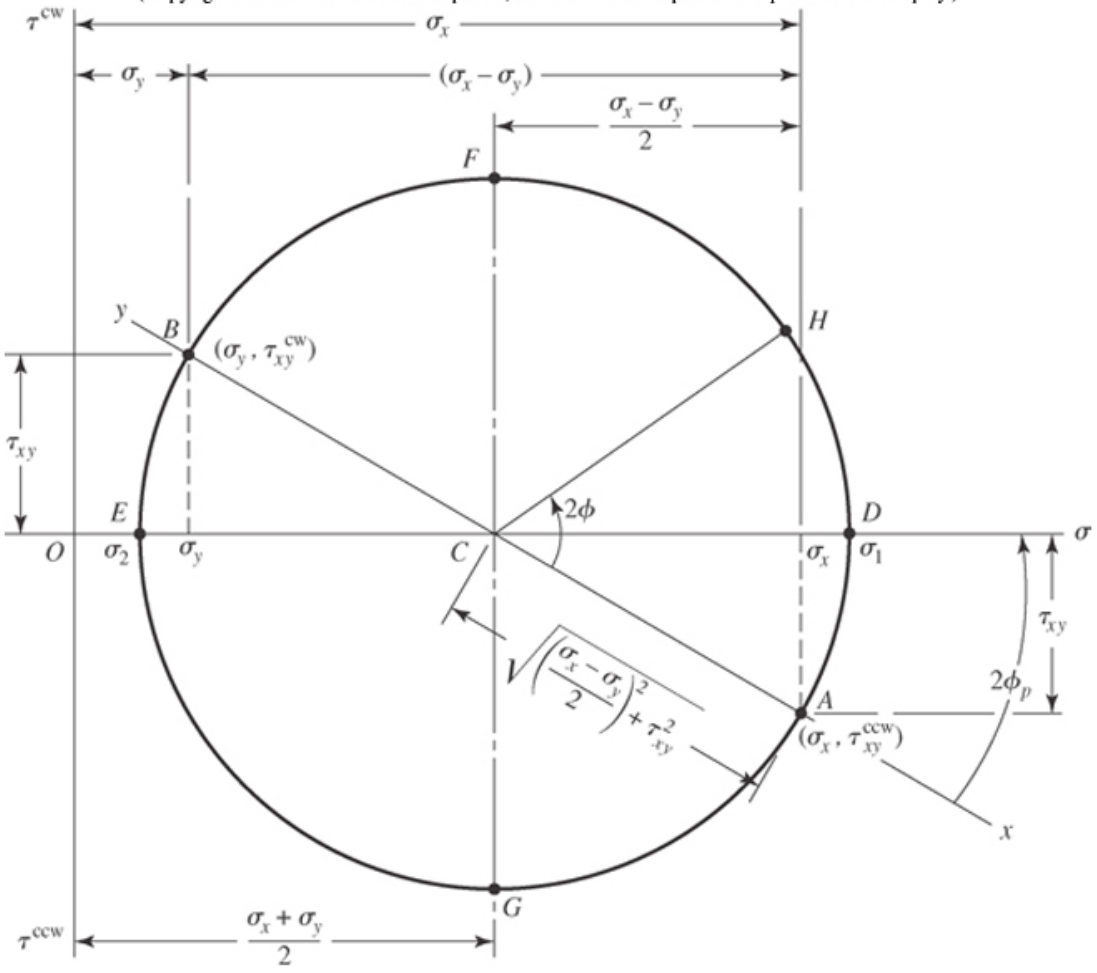

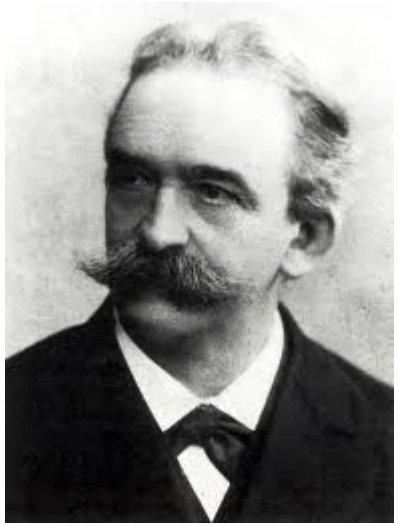

Christian Otto Mohr
German civil engineer
(October 8, 1835 – October 2, 1918)

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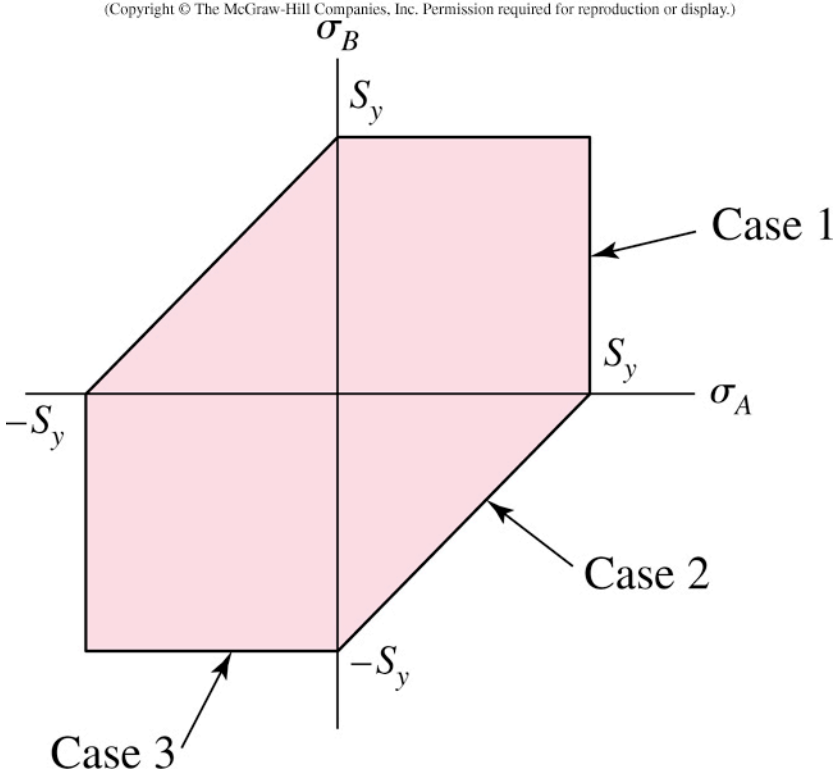

Henri Édouard Tresca
French mechanical engineer
(October 12, 1814–June 21, 1885)

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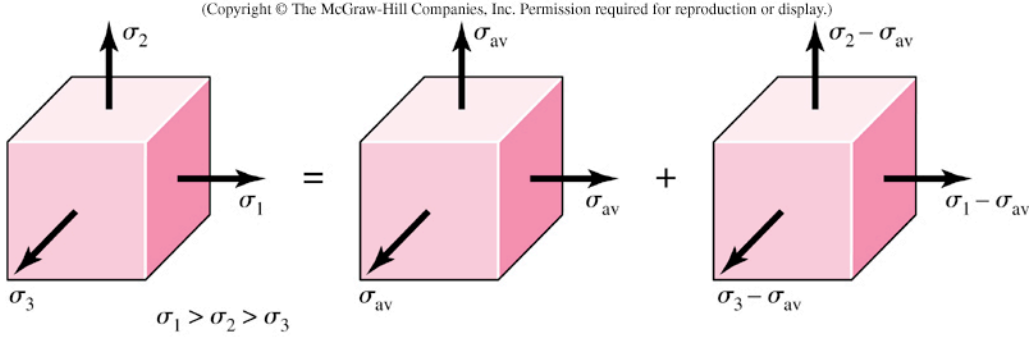

(*a*) Triaxial stresses (*b*) Hydrostatic component (*c*) Distortional component

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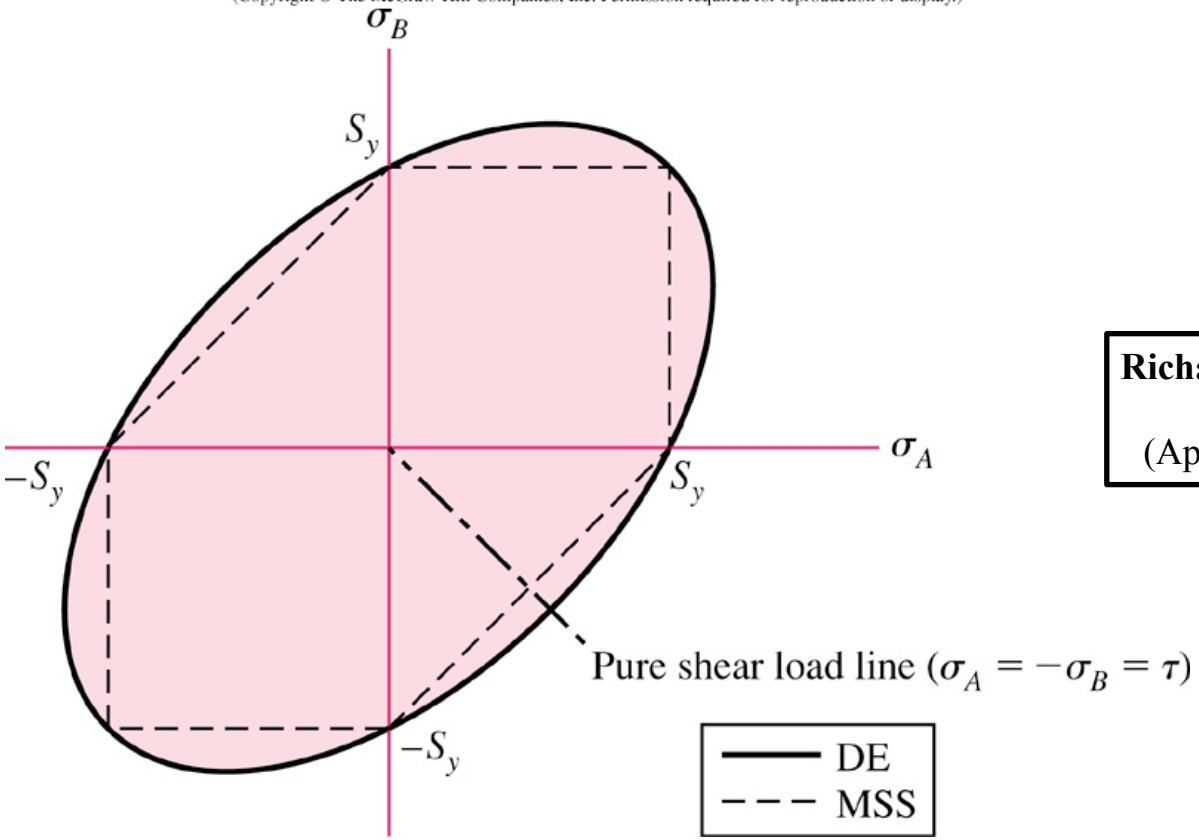

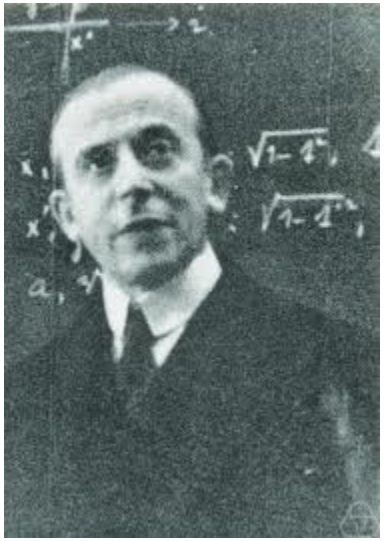

Richard Martin Elder von Mises
Austeria Economics
(April 19, 1883 – July 14, 1953)

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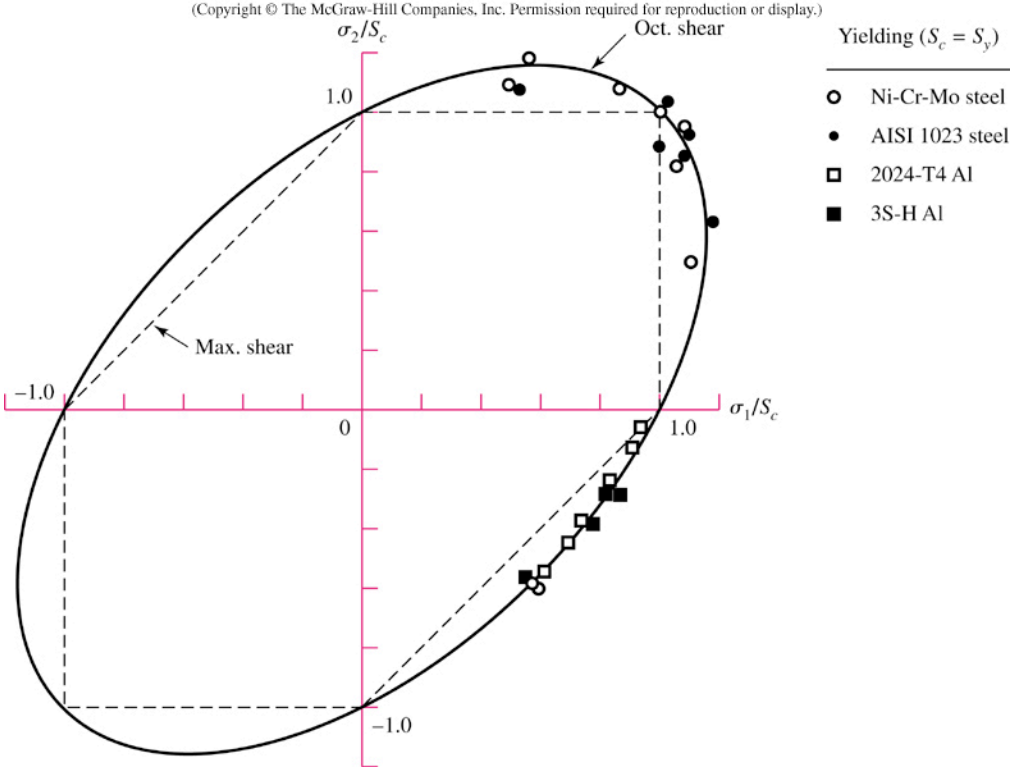

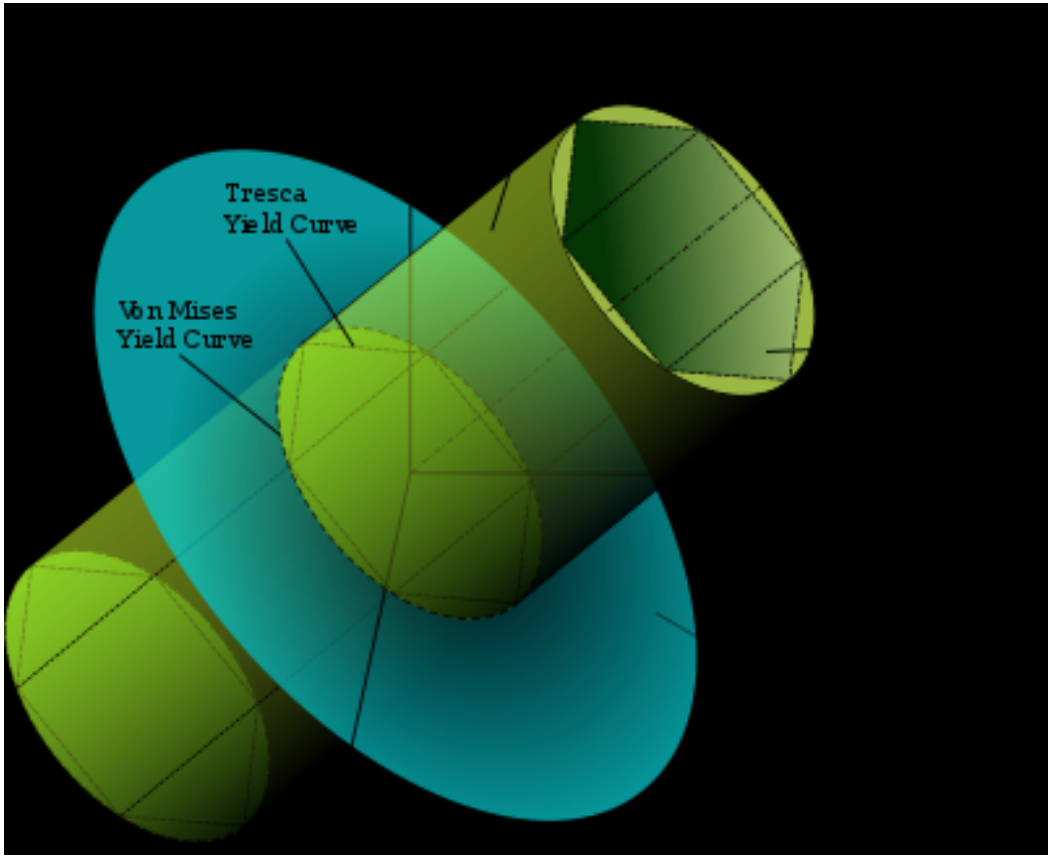

$$\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right] \leq \ \sigma_y^2.$$

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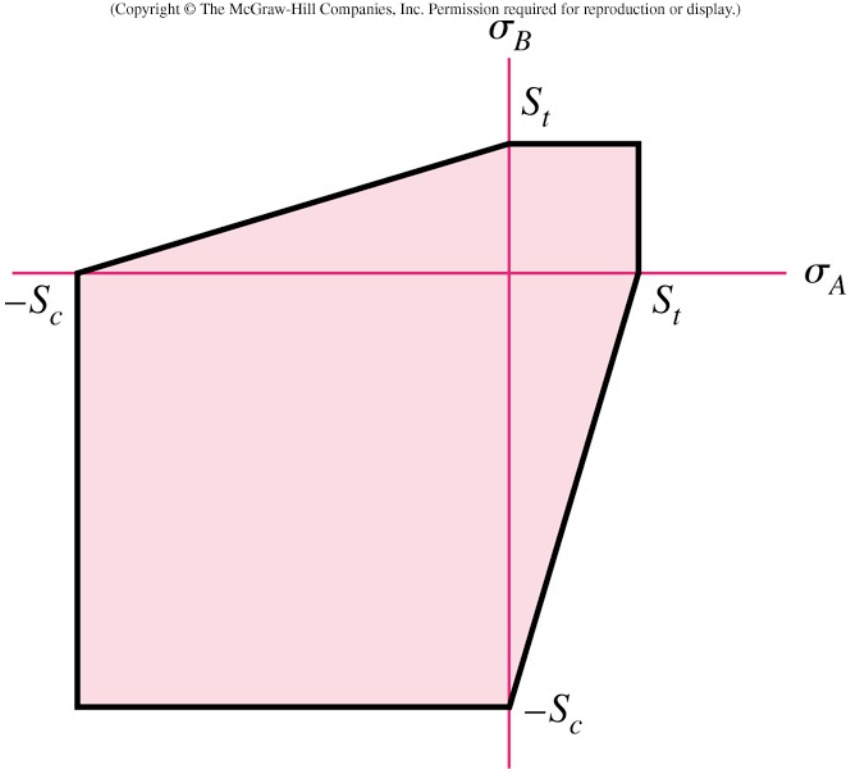
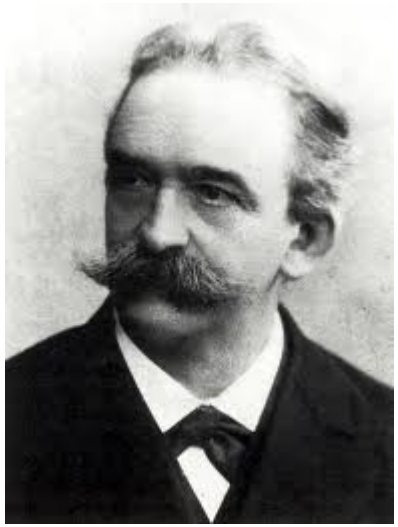


Charles Augustin de Coulomb
French (1736 - 1806)



Christian Otto Mohr

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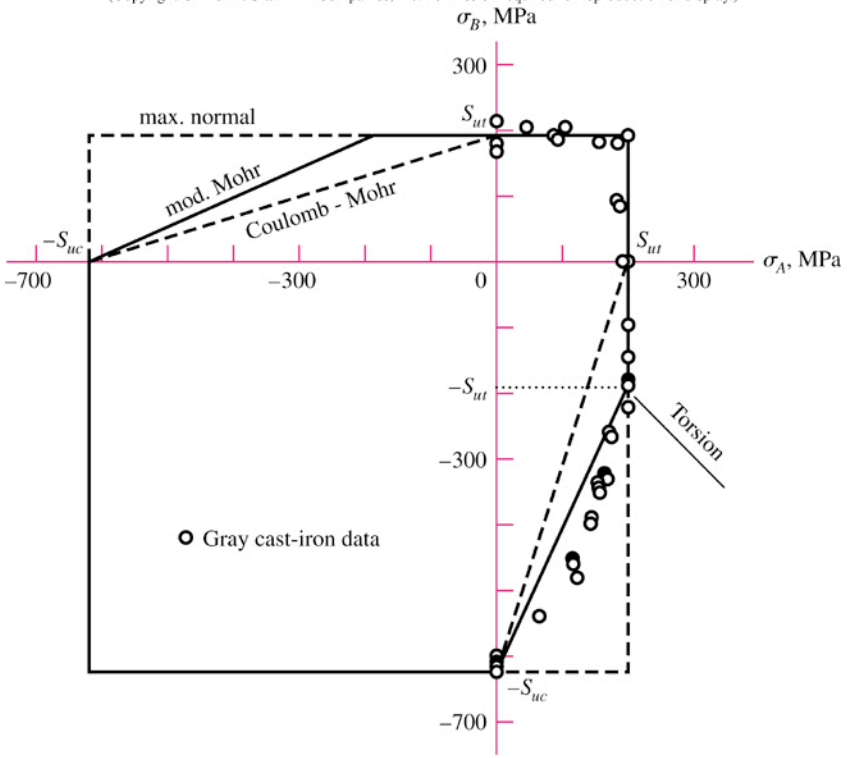

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