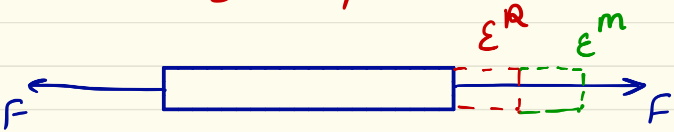
Composites — Lesson 14 — بسم الله الرحمن الرحيم

4.8 - Hygroscopic Stress Analysis



$$\{\varepsilon\} = \{\varepsilon\}^R + \{\varepsilon\}^M$$

$$\Delta_m = \beta \, \ell \, \Delta C$$

ΔC → humidity change

β → Coefficient of moisture expansion (CME)

$$\{\sigma\}_k = [\bar{Q}]_k \left(\{\varepsilon\} - \{\varepsilon\}^M \right)$$

orthotropic materials

$$\begin{Bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{Bmatrix} \rightarrow \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy}/2 \end{Bmatrix} = [T(\theta)]^{-1} \begin{Bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{Bmatrix}$$

$$\left\{\begin{array}{c}\bar{N} \\ \hdashline \bar{M}\end{array}\right\} = \left[\begin{array}{c:c} A & B \\ \hdashline B & D \end{array}\right] \left\{\begin{array}{c}\varepsilon^{\circ} \\ \hdashline K\end{array}\right\}$$

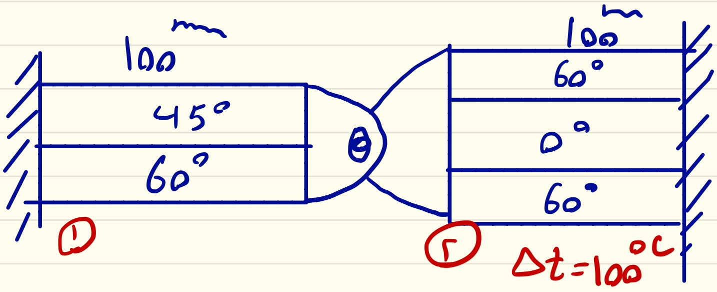
$$\{\bar{N}\} = \{N\} + \{N\}^{t} + \{N\}^{m}$$

$$\{\bar{M}\} = \{M\} + \{M\}^{t} + \{M\}^{m}$$

$$\{N\}^{m} = \sum_{k=1}^{N} [\bar{Q}]_{k} \{\beta\}_{k} \Delta C \,(z_{k} - z_{k-1})$$

$$\{M\}^{m} = \sum_{k=1}^{N} [\bar{Q}]_{k} \{\beta\}_{k} \Delta C \,\tfrac{1}{2}(z_{k}^{2} - z_{k-1}^{2})$$

Exercise: Find the stresses in laminate number 2.



$N_{x1} = N_{x2}$

$N_{y1} = N_{xy1} = 0$

$N_{y2} = N_{xy2} = 0$

all thicknesses: 5 mm

$$[Q] = \begin{bmatrix} 20 & 0.7 & 0 \\ 0.7 & 0.2 & 0 \\ 0 & 0 & 0.7 \end{bmatrix} \ GN/m^2$$

4.9_ Laminate Strength_Analysis Procedure

1- calculate laminate Stiffnesses matrices A, B, D

2- calculate termal and hygroscopic effect on force and moment (N^t, M^t, N^m, M^m)

3- Reduce the load and bring it to 1/10 of the actual load. (if we have load and we want to find the effect of it)

4- $\{\bar{N}\} = \{N\} + \{N\}^t + \{N\}^m \quad , \quad \{\bar{M}\} = \{M\} + \{M\}^t + \{M\}^m$

5- Use EQS to find the strains and curvatures:

$$\begin{Bmatrix} \bar{N} \\ \bar{M} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^{\circ} \\ K \end{Bmatrix} \Rightarrow \begin{Bmatrix} \varepsilon^{\circ} \\ K \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{Bmatrix} \bar{N} \\ \bar{M} \end{Bmatrix}$$

6- Find strains and stresses in each layer (in the middle of them)

$$\{\sigma\}_k = [\bar{Q}]_k \, [\underbrace{(\{\varepsilon^{o}\} + z\{k\}) - \{\varepsilon\}^{t} - \{\varepsilon\}^{m}}_{\{\varepsilon\}^{R}_{k}}]$$

7- Find the stresses in the principle orthotropy directions.

$$\{\sigma_1\}_k = [T(\theta)]\{\sigma_x\}_k$$

8- Apply failure criterion for each layer

for example

$$R_1 = \frac{\sigma_1}{X_t} \text{ or } \frac{\sigma_1}{X_c} \leq 1$$

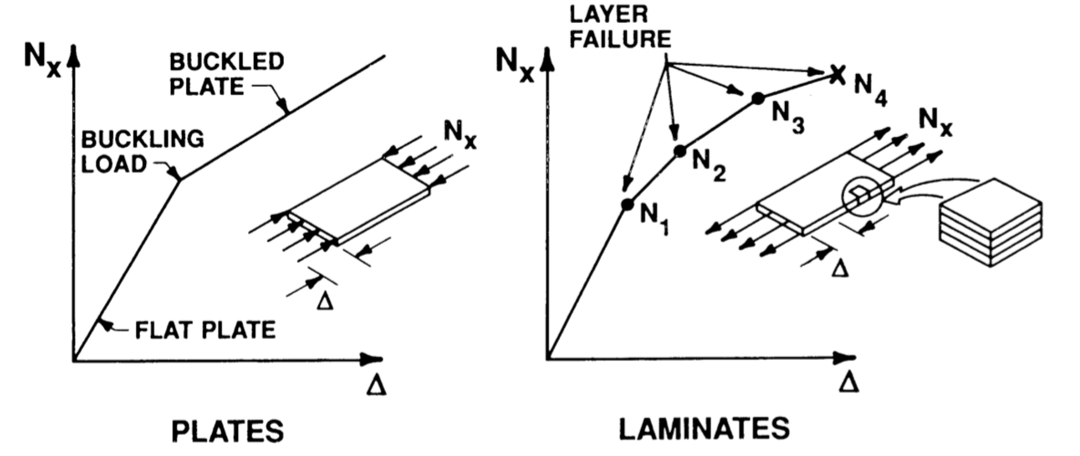
$$R_2 = \frac{\sigma_2}{Y_t} \text{ or } \frac{\sigma_2}{Y_c} \leq 1$$

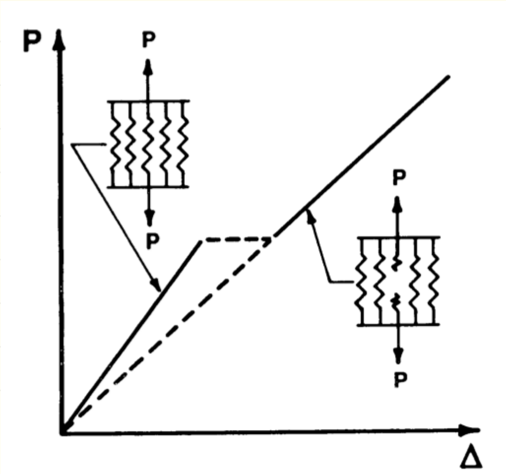
$$R_3 = \frac{\tau_{12}}{S} \leq 1$$

9- If you have failure in one or more laminae, degrade the failed laminae by setting the engineering constants to zero. calculate A, B, D again and find stresses in each layer in the new situation.

10- Apply failure criterion for each layer again. If you do not have failed layer, you can increas the applied force by %10.

11- At the end, if no layer left, that force is the strength of the laminate.


N_x
BUCKLED PLATE
BUCKLING LOAD
N_x
FLAT PLATE
Δ
Δ
PLATES
LAYER FAILURE
N_x
N_4
N_3
N_x
N_2
N_1
Δ
Δ
LAMINATES


P
P
P
P
P
Δ

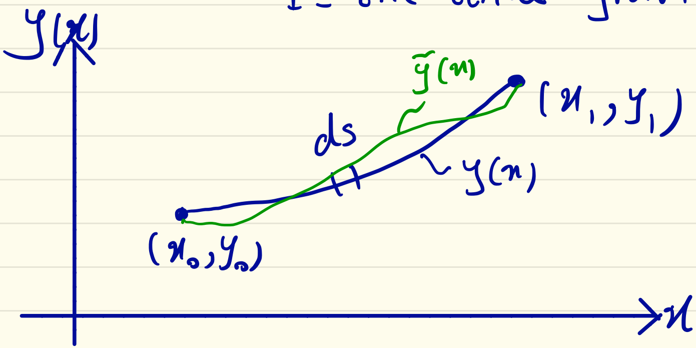
Chapter 5_ Preliminary Remarks for Advanced mechanics of composites

5.1_ Calculus of variation

Consider a rope hanging between two points (x_0, y_0) and (x_1, y_1)

Let ρ = the mass density of the rope (mass per unite length)

I = the total gravitational potential energy of the rope



$$I = \int y g \rho \, ds = \int_{x_0}^{x_0} y \rho g \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx \qquad (5.1\text{-}1)$$

$$I \equiv \int_{x_0}^{x_1} F(y, y')\,dx \qquad (5.1\text{-}2)$$

The "principle of minimum total potential energy" tells us that when the rope is in equilibrium, its total potential energy is minimum. In other words, for any other "admissible function" $\tilde{y}(x)$ we will have

$$I(\tilde{y}) - I(y) \geq 0 \qquad (5.1\text{-}3)$$

The minimizing function $y(x)$ is called the "extermal (function)". An extermal (function) may also be a maximizing function in some problems. (stationary)

In the language of mathematics, $I(y)$ is called a "functional" which, in general, a functional is a function of functions.

In a typical variational problem we are looking for the extermal of a functional. (Stationary function)

In dynamical problems "Hamiltons" principle tells us that the actual dynamical path $q(t)$ will make the functional

$$I(q) = \int_{t_0}^{t_1} L(q, \dot{q})\, dt$$

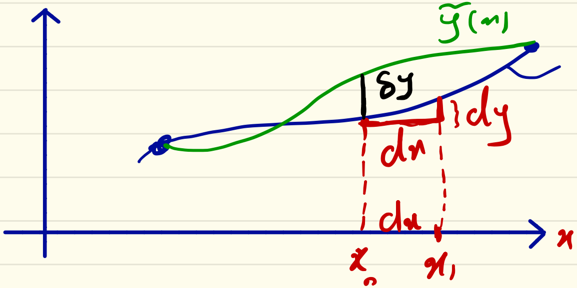
extermum.

$$L = T - V$$

T = kinetic energy, V = gravitational potential energy

The variation of a functional (here "I") is defined as follow

$$\boxed{\delta I = I(\tilde{y}) - I(y)} \qquad (5.1\text{-}4)$$



$$\delta y = \tilde{y} - y$$

error

In calculus of variation it can be proved that at the exermum of $I(\tilde{y})$, the variation of I must vanish:

$$\boxed{I(y) \text{ is stationary} \Longleftrightarrow \delta I = 0} \qquad (5.1\text{-}5)$$