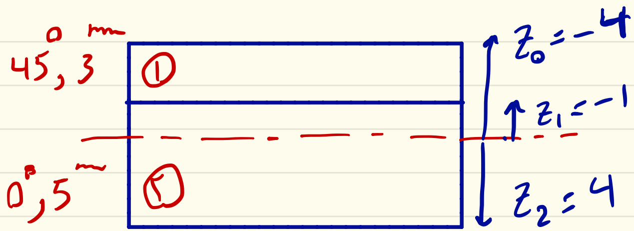
Composites Lesson 13 بسم الله الرحمن الرحيم

Exampel 1: Consider a two-layered laminate. The bottom layer is zero degree and its thickness is 5 mm. The top layer has an angle of 45° and a thickness of 3 mm. Find the stiffness matrices of laminate. The stiffnes matrix of the material used is as follow:



$$[Q] = \begin{bmatrix} 20 & 0.7 & 0 \\ 0.7 & 0.2 & 0 \\ 0 & 0 & .7 \end{bmatrix} \frac{GN}{m^2}$$

$$[A] = \sum_{K=1}^{N} [\bar{Q}]_K (z_K - z_{K-1}) = [\bar{Q}]_{45^\circ}\big((-1)-(-4)\big)$$

$$+ [\bar{Q}]_{0^\circ}\big(4-(-1)\big)$$

$$= 3\begin{bmatrix} 6.55 & 5.15 & 4.5 \\ & 6.55 & 4.5 \\ \text{sym} & & 5.15 \end{bmatrix} + 5\begin{bmatrix} 20 & 0.7 & 0 \\ & 0.2 & 0 \\ \text{sym} & & 0.7 \end{bmatrix} = \begin{bmatrix} 119.65 & 18.95 & 13.5 \\ & 29.65 & 13.5 \\ \text{sym} & & 18.95 \end{bmatrix}$$

$$B = \sum_{K=1}^{N} [\bar{Q}]_K \, \tfrac{1}{2}\left(z_K^2 - z_{K-1}^2\right) = \tfrac{1}{2}[\bar{Q}]_{45^\circ}\left((-1)^2 - (-4)^2\right) + \tfrac{1}{2}[\bar{Q}]_{0^\circ}\left(4^2 - (-1)^2\right)$$

$$= 7.5\left\{ -\begin{bmatrix} 6.55 & 5.15 & 4.5 \\ & 6.55 & 4.5 \\ \text{sym} & & 5.15 \end{bmatrix} + \begin{bmatrix} 20 & 0.7 & 0 \\ & 0.2 & 0 \\ \text{sym} & & 0.7 \end{bmatrix} \right.$$

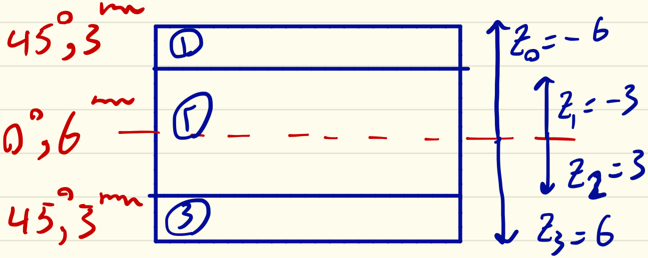
$$= \begin{bmatrix} 100.9 & -38.4 & -33.75 \\ & -34.1 & -33.75 \\ \text{sym} & & -33.4 \end{bmatrix} \text{GPa.mm}^2$$

GPa.mm^3

$$D = \sum_{K=1}^{N} [\bar{Q}]_K \, \tfrac{1}{3}\left(z_K^3 - z_{K-1}^3\right) = \dots = \begin{bmatrix} 571 & 123 & 94.5 \\ & 181 & 94.5 \\ \text{sym} & & 123 \end{bmatrix}$$

Example 2: In a three-layered laminate the top layers have an angle of 45° and a thickness of 3 mm. The middle layer is 0° and 6 mm thick. The properties of material are as in Example 1. Find the stiffness matrices of the laminate.

$$[\bar{Q}]_2 = \begin{bmatrix} 20 & 0.7 & 0 \\ & 0.2 & 0 \\ \text{sym} & & 0.7 \end{bmatrix} \text{ GPa}$$



$$[\bar{Q}]_1 = [\bar{Q}]_3 = \begin{bmatrix} 6.55 & 5.15 & 4.5 \\ & 6.55 & 4.5 \\ \text{sym} & & 5.15 \end{bmatrix} \text{ GPa}$$

$$[A] = \sum_{K=1}^{N} [\bar{Q}]_K (z_K - z_{K-1}) = [\bar{Q}]_1 \big((-3)-(-6)\big) + [\bar{Q}]_2 \big(3-(-3)\big) + [\bar{Q}]_3 (6-3)$$

$$A = \begin{bmatrix} 159.3 & 35.1 & 27 \\ & 51.3 & 27 \\ \text{sym} & & 35.1 \end{bmatrix}$$

$$B=\sum_{k=1}^{N}[\bar{Q}]_k \tfrac{1}{2}\left(z_k^2-z_{k-1}^2\right)=\cancel{\tfrac{1}{2}[\bar{Q}]_1\left((-3)^2-(-6)^2\right)}$$

$$+\tfrac{1}{2}[\bar{Q}]_2\left(\underbrace{3^2-(-3)^2}_{0}\right)+\cancel{\tfrac{1}{2}[\bar{Q}]_3\left(6^2-3^2\right)}=0$$

$$D=\sum_{k=1}^{N}[\bar{Q}]_k \tfrac{1}{3}\left(z_k^3-z_{k-1}^3\right)=\cdots=\begin{bmatrix}1185.3 & 661.3 & 567\\ & 861.3 & 567\\ \text{sym} & & 661.5\end{bmatrix}$$

$GPa.mm^3$

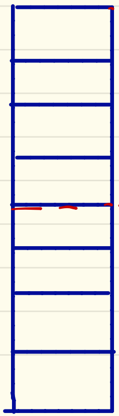
Example 3: Consider 8 laminae with 45° fiber orientation and 3^{mm} thickness. The properties of material are the same as in previous Examples. Find the Stiffness matrices of the laminates for the following states:

a- All the layers have the same angle $+45^\circ$

b- $[(45)_2/(-45)_2]_s$

c- $[(\pm 45)_2]_s$

d- $[\pm\mp 45]_s$



b	c	d
+45	+45	+45
+45	-45	-45
-45	+45	-45
-45	-45	+45
-45	-45	+45
-45	+45	-45
+45	-45	-45
+45	+45	+45

(a)

$$A = \begin{bmatrix} 157.2 & 123.6 & 108 \\ & 157.2 & 108 \\ \text{sym} & & 123.6 \end{bmatrix} \text{GPa.mm}, \quad [B] = 0, \quad [D] = \begin{bmatrix} 7.55 & 5.93 & 5.18 \\ & 7.55 & 5.18 \\ \text{sym} & & 5.93 \end{bmatrix}$$

$$\frac{D_{16}}{D_{11}} = 0.686$$

(b)

$$[A] = \begin{bmatrix} 157.2 & 123.6 & 0 \\ & 157.2 & 0 \\ \text{sym} & & 123.6 \end{bmatrix}, \quad [B] = 0, \quad [D] = \begin{bmatrix} 7.55 & 5.93 & 3.89 \\ & 7.55 & 3.89 \\ \text{sym} & & 5.93 \end{bmatrix}$$

$$\frac{D_{16}}{D_{11}} = 0.515$$

(c)

$$[A] = [\text{previous}], \quad [B] = 0, \quad [D] = 10^3 \begin{bmatrix} 7.55 & 5.93 & 1.94 \\ & 7.55 & 1.94 \\ & & 5.93 \end{bmatrix} \text{GPa.mm}^3$$

$$\frac{D_{16}}{D_{11}} = 0.257$$

(d)

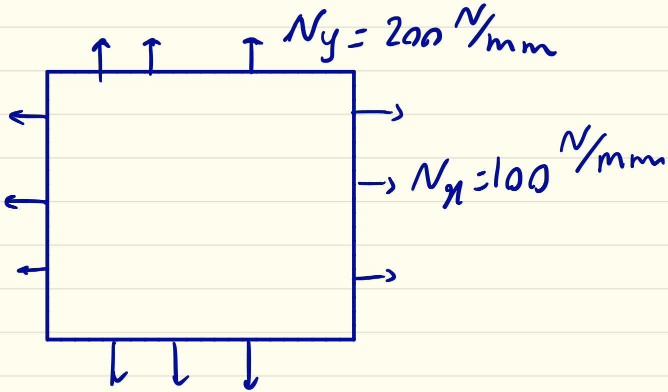
$[A] = [\text{previous}]$, $[B] = 0$,

$$[D] = \begin{bmatrix} 7.55 & 5.93 & 0.97 \\ & 7.55 & 0.97 \\ \text{sym} & & 5.93 \end{bmatrix}$$

$$\frac{D_{16}}{D_{11}} = 0.129$$

[A] does not depend on how the layers are arranged but the components D_{16} & D_{26} change as the layers arranging changes.

Example 4: Consider tha laminate in Example 2 by loading below and find the stress in each layer.



$$\{N\} = [A]\{\varepsilon\}^0 \Rightarrow \{\varepsilon\}^0 = [A]^{-1}\{N\}$$

$$\underbrace{\{M\}}_{0} = [D]\{K\} \Rightarrow \{K\} = \{0\}$$

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = 10^{-3} \begin{bmatrix} 0.00759 & -0.00356 & -0.00309 \\ & 0.03441 & -0.02373 \\ \text{sym} & & 0.04911 \end{bmatrix} \begin{Bmatrix} 100 \\ 200 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} 0.00685 \\ 0.00332 \\ -0.00784 \end{Bmatrix}$$

$$\{\sigma\}_k = [\bar{Q}]_k \Big(\underbrace{\{\varepsilon\}^{0} + z\{\cancel{\kappa}\}^{\,0}}_{\{\varepsilon\}} \Big)$$

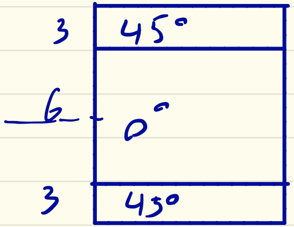
0° layer:

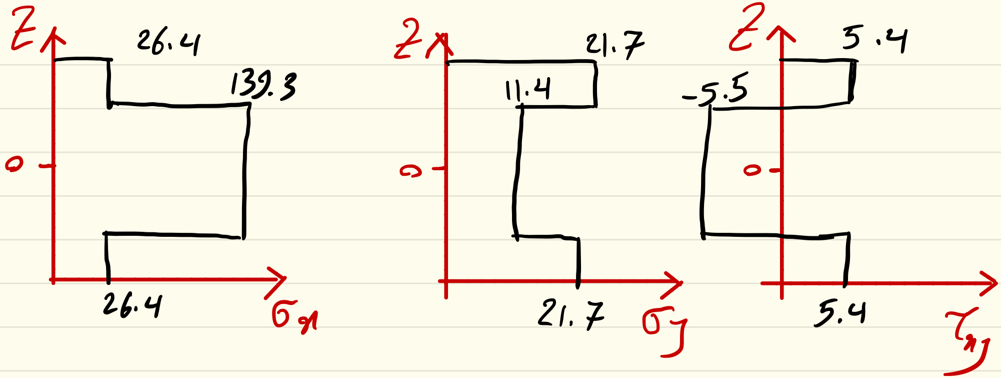
$$\{\sigma\}_{0^\circ} = \begin{bmatrix} 20 & 0.7 & 0 \\ & 0.2 & 0 \\ \text{sym} & & 0.7 \end{bmatrix} \begin{Bmatrix} 0.00685 \\ 0.00332 \\ -0.00784 \end{Bmatrix} = \begin{Bmatrix} 139.3 \\ 11.4 \\ -5.5 \end{Bmatrix} \times 10^{-3} \ GN/m^2$$

45° layers:

$$\{\sigma\}_{45^\circ} = \begin{bmatrix} 6.53 & 5.3 & 4.5 \\ & 6.55 & 4.5 \\ \text{sym} & & 5.15 \end{bmatrix} \begin{Bmatrix} 0.00685 \\ 0.00332 \\ -0.00784 \end{Bmatrix}$$

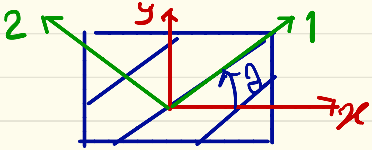
$$= \begin{Bmatrix} 26.4 \\ 21.7 \\ 5.4 \end{Bmatrix} \times 10^{-3} \ \frac{GN}{m^2}$$





The stresses in principal material (orthotropy) direction:

$$\{\sigma_1\}_{0^\circ} = \{\sigma_x\}_{0^\circ}$$



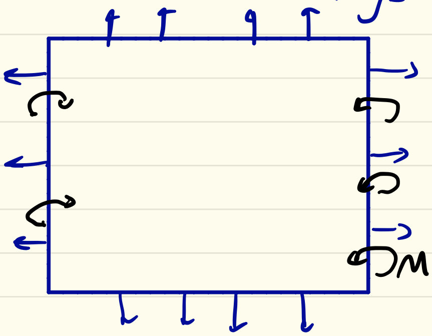
$45^\circ = ?$

$$[T] = \begin{bmatrix} 0.5 & 0.5 & 1 \\ 0.5 & 0.5 & 1 \\ -0.5 & 0.5 & 0 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}_{45^\circ} = \begin{bmatrix} 0.5 & 0.5 & 1 \\ 0.5 & 0.5 & -1 \\ -0.5 & 0.5 & 0 \end{bmatrix} \begin{Bmatrix} 26.4 \\ 21.7 \\ 5.4 \end{Bmatrix} = \begin{Bmatrix} 29.6 \\ 18.8 \\ -2.5 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{Bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 1 \\ 0.5 & 0.5 & -1 \\ -0.5 & 0.5 & 0 \end{bmatrix} \begin{Bmatrix} 0.00685 \\ 0.00332 \\ \frac{-0.00784}{2} \end{Bmatrix} = \begin{Bmatrix} 0.00116 \\ 0.009 \\ -0.00176 \end{Bmatrix}$$

Example 5: Solve the same laminate of previous example by the following loads:



$N_y = 200\ N/mm$

$N_x = 100\ N/mm$

$M_x = 100\ \frac{N.m}{mm}$

$$\{N\} = \{A\}\{\varepsilon\}^0 \longrightarrow \{\varepsilon\}^0 = [A]^{-1}\{N\}$$

$$\{M\} = [D]\{K\} \longrightarrow \{K\} = [D]^{-1}\{M\}$$

$$\{\varepsilon\}^0 = \{\text{Previous}\} = \begin{Bmatrix} 0.00658 \\ 0.00332 \\ -0.00784 \end{Bmatrix}$$

$$\{K\} = \begin{Bmatrix} 0.162 \\ -0.076 \\ -0.074 \end{Bmatrix} \times 10^{-3}$$

$$\{\sigma\}_k = [\bar{Q}]_k \left(\{\varepsilon\}^0 + z\{K\} \right)$$

The top 45° layer: $z=-6$ to -3

$$z=-6 \Rightarrow \{\sigma\}_{45^\circ} = [\bar{Q}]_{45^\circ}\left(\{\varepsilon\}^0 - 6\{\kappa\}\right) = \begin{Bmatrix} 0.1207 \\ 0.0117 \\ -0.0052 \end{Bmatrix}$$

$$z=-3 \Rightarrow \{\sigma\}_{45^\circ} = [\bar{Q}]_{45^\circ}\left(\{\varepsilon\}^0 - 3\{\kappa\}\right) = \begin{Bmatrix} 0.1303 \\ 0.0116 \\ -0.0053 \end{Bmatrix}$$

The 0° layer: $z=-3$ to $+3$

$$z=-3 \longrightarrow \{\sigma\}_{0^\circ} = \begin{Bmatrix} 0.0258 \\ 0.0218 \\ 0.0055 \end{Bmatrix}$$

$$z=+3 \longrightarrow \{\sigma\}_{0} = \begin{Bmatrix} 0.0279 \\ 0.0219 \\ 0.0055 \end{Bmatrix}$$