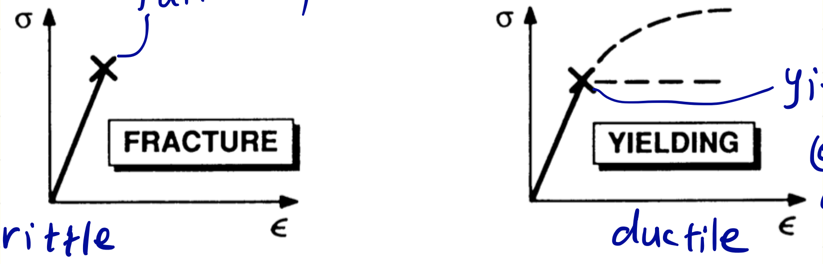
Composites Lesson 9 بسم الله الرحمن الرحيم

## 3.9-2- Biaxial Strength Criteria for an Orthotropic Lamina

The general practical problem involves at least a biaxial if not a triaxial state of stresses. Thus, a logical method of using uniaxial strength information is required for analysis of multiaxial loading problems.

failure point (surface/envelope)

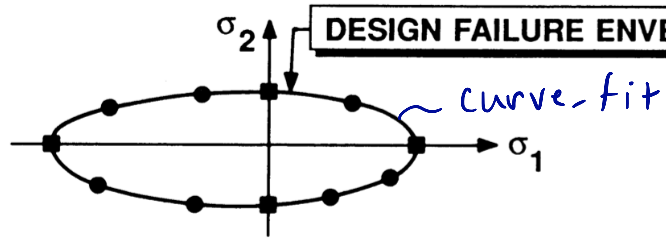



yield point (end of linear elastic behavior)

brittle

ductile

Figure 2-32 Failure (Fracture) versus Yielding our objective is to find the analytical definition of the failure surface or envelope in stress space.



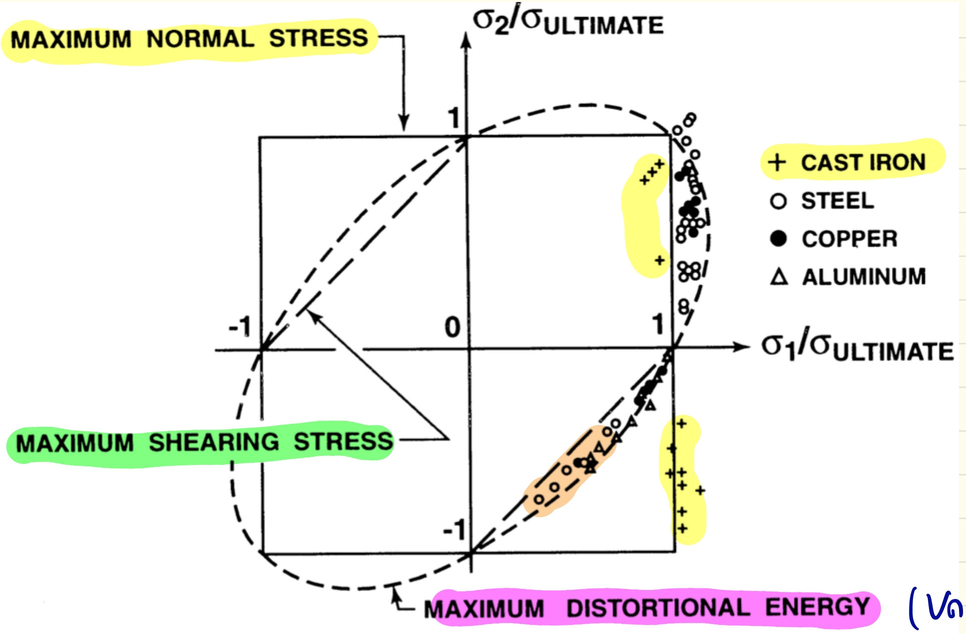

e 2-33 Hypothetical Two-Dimensional Failure Data and Design Curve

1- Analytical

2- curve-fit

curve-fit criteria are generally disassociated from knowledg of precisely how the material fails.

Isotropic Materials




(Von Mises)

Different metals fail in different manners and thereby require different failure criteria.

For Composite materials:

① maximum stress failure criterion

$\sigma_1, \sigma_2, \tau_{12}$: stresses in principal material coordinates

$$\begin{cases} \sigma_1 < X_t & \text{Tension} \\ \sigma_1 > X_c & \text{Compression} \end{cases}$$

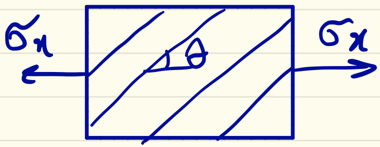
$$\begin{cases} \sigma_2 < Y_t & \text{Ten.} \\ \sigma_2 > Y_c & \text{Com.} \end{cases}$$

If any one of these inequalities is not satisfied, then the material has failed.

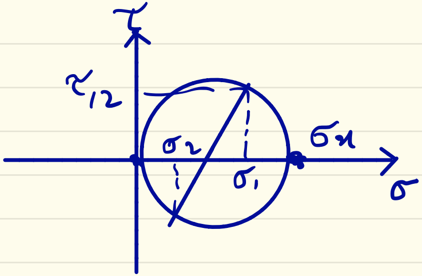
$$|\tau_{12}| < S$$

Note that there is no interaction between modes of failure in this criterion.

Example:




$$\begin{cases} \sigma_1 = \sigma_x \cos^2\theta \\ \sigma_2 = \sigma_x \sin^2\theta \\ \tau_{12} = -\sigma_x \sin\theta \cos\theta \end{cases}$$




$$\begin{cases} \dfrac{X_c}{\cos^2\theta} < \sigma_x < \dfrac{X_t}{\cos^2\theta} \\[2ex] \dfrac{Y_c}{\sin^2\theta} < \sigma_x < \dfrac{Y_t}{\sin^2\theta} \\[2ex] |\sigma_x| < \left| \dfrac{S}{\sin\theta \cos\theta} \right| \end{cases}$$

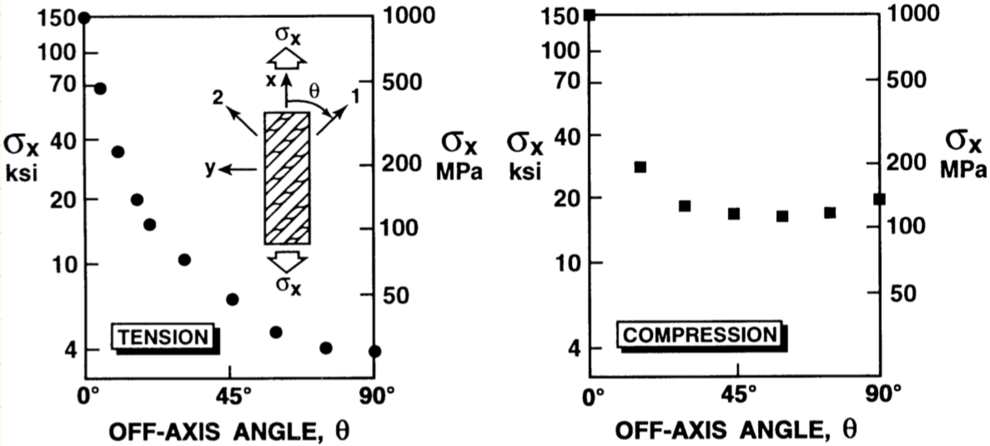
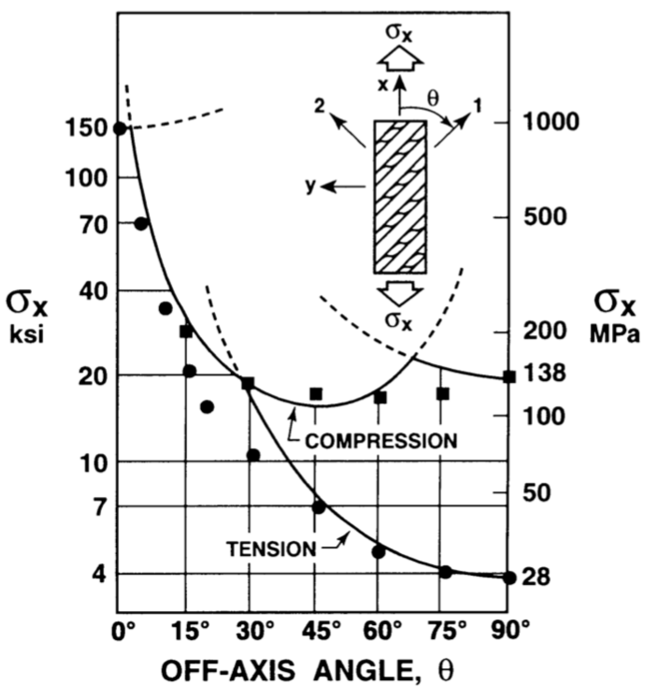



Figure 2-36 Measured Failure Data for Glass-Epoxy *(After Tsai [2-21])*

Uniaxial Strength test




37 Maximum Stress Failure Criterion *(After T*

## ② Maximum Strain Failure Criterion

$X_{\varepsilon t}$ ($X_{\varepsilon c}$) = maximum tensile (compressive) normal strain in the 1-direction

$Y_{\varepsilon t}$ ($Y_{\varepsilon c}$) = maximum tensile (compressive) normal strain in the 2-direction

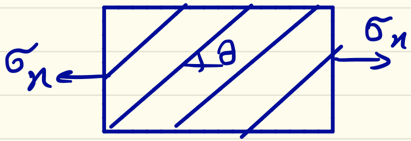
$S_{\varepsilon}$ = maximum shear strain in the 1-2 Coordinates

$$\begin{cases} \varepsilon_1 < X_{\varepsilon t} \\ \varepsilon_1 > X_{\varepsilon c} \end{cases}$$

$$\begin{cases} \varepsilon_2 < Y_{\varepsilon t} \\ \varepsilon_2 > Y_{\varepsilon c} \end{cases}$$

$$|\gamma_{12}| < S_{\varepsilon}$$

Example

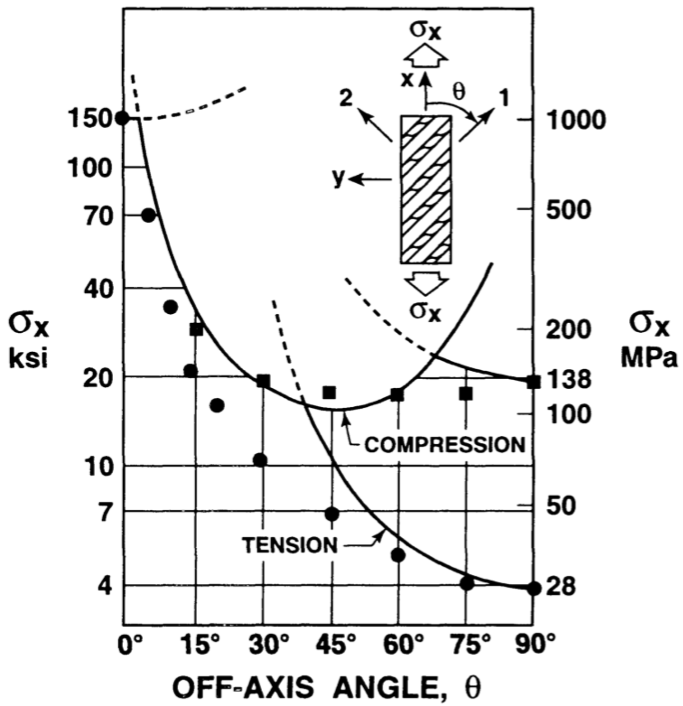



$$\begin{cases} \sigma_1 = \sigma_x \cos^2\theta \\ \sigma_2 = \sigma_x \sin^2\theta \\ \tau_{12} = -\sigma_x \sin\theta \cos\theta \end{cases}$$

$$\begin{cases} \varepsilon_1 = \frac{1}{E_1}(\sigma_1 - \nu_{12}\sigma_2) \\ \varepsilon_2 = \frac{1}{E_2}(\sigma_2 - \nu_{12}\sigma_1) \\ \gamma_{12} = \tau_{12}/G_{12} \end{cases} \Rightarrow \begin{cases} \varepsilon_1 = \frac{1}{E_1}(\cos^2\theta - \nu_{12}\sin^2\theta)\sigma_x \\ \varepsilon_2 = \frac{1}{E_2}(\sin^2\theta - \nu_{12}\cos^2\theta)\sigma_x \\ \gamma_{12} = -\frac{1}{G_{12}}(\cos\theta \sin\theta)\sigma_x \end{cases}$$

$$\begin{cases} X_{\varepsilon t} = \frac{X_t}{E_1} \\ X_{\varepsilon c} = \frac{X_c}{E_1} \end{cases} \qquad \begin{cases} Y_{\varepsilon t} = \frac{Y_t}{E_2} \\ Y_{\varepsilon c} = \frac{Y_c}{E_2} \end{cases} \qquad S_\varepsilon = \frac{S}{G_{12}}$$

$$
\begin{cases}
\dfrac{X_c}{\cos^2\theta - \nu_{12}\sin^2\theta} < \sigma_x < \dfrac{X_t}{\cos^2\theta - \nu_{12}\sin^2\theta} \\[2ex]
\dfrac{Y_c}{\sin^2\theta - \nu_{12}\cos^2\theta} < \sigma_x < \dfrac{Y_t}{\sin^2\theta - \nu_{12}\cos^2\theta} \\[2ex]
|\sigma_x| < \left| \dfrac{S}{\cos\theta \sin\theta} \right|
\end{cases}
$$



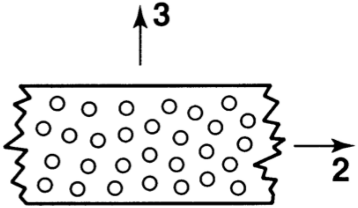

2-38 Maximum Strain Failure Criterion *(After Tsai [*

## ③ Tsai-Hill Failure Criterion

Hill's criterion is an extension of Von Mises' yield criterion.

$$(G+H)\sigma_1^2 + (F+H)\sigma_2^2 + (F+G)\sigma_3^2 - 2H\sigma_1\sigma_2 - 2G\sigma_1\sigma_3 - 2F\sigma_2\sigma_3 + 2L\tau_{23}^2 + 2M\tau_{13}^2 + 2N\tau_{12}^2 = 1$$

F, G, H, L, M, N will be regarded as failure strength.




Cross Section of a Unidirectional Lamina with Fibers in the 1-Direction

For a lamina that $Z = Y$ we can rewrite it as:

$$\boxed{\frac{\sigma_1^2}{X^2} - \frac{\sigma_1\sigma_2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\tau_{12}^2}{S^2} = 1}$$

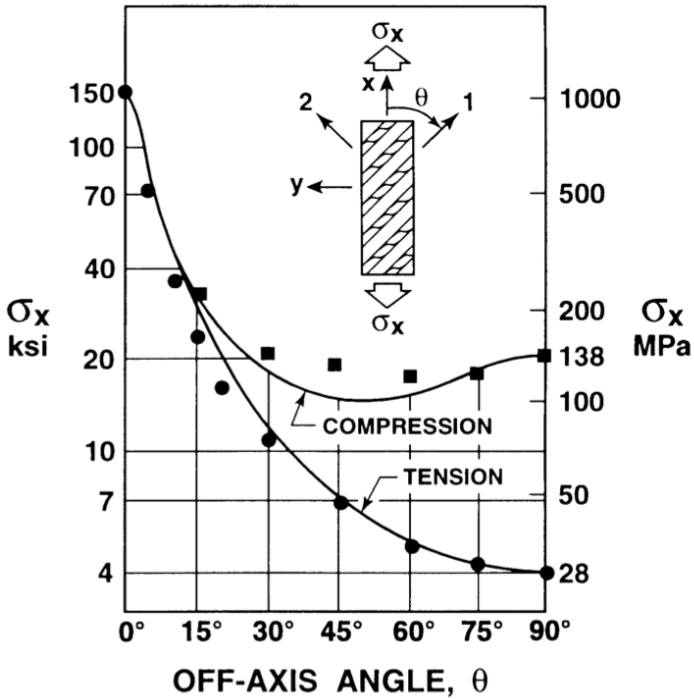
The appropriate values of $X_t$ or $X_c$ and $Y_t$ or $Y_c$ must be used depending on the signs of $\sigma_1$ and $\sigma_2$.

It is one criterion, not three as in previous failure criteria.

Example: uniaxial off-axis strength

$$\frac{\cos^4\theta}{X^2} + \left[\frac{1}{S^2} - \frac{1}{X^2}\right]\cos^2\theta\,\sin^2\theta + \frac{\sin^4\theta}{Y^2} = \frac{1}{\sigma_x^2}$$

E-glass epoxy




e 2-40 Tsai-Hill Failure Criterion *(After Tsai [2-2*

(1) The variation of strength with angle of lamina orientation is smooth rather than having cusps that are not seen in experimental results.

(2) The strength continuously decreases as $\theta$ grows from 0° rather than the rise in uniaxial strength that is characteristic of both the maximum stress and the maximum strain criteria.

(3) The agreement between the criterion and experiment is even better than that at first glance because Figures 2-37, 2-38, and 2-40 are plotted at a logarithmic scale. The maximum stress and strain criteria are incorrect by 100% at 30°!

(4) Considerable *interaction* exists between the failure strengths X, Y, S in the Tsai-Hill criterion, but *none* exists in the previous criteria where axial, transverse, and shear failures are presumed to occur independently.

## ④ Hoffman Failure Criterion

To accout for different strengths in tension and compression Hoffman added linear terms to Hill's equation.

$$C_1(\sigma_2-\sigma_3)^2+C_2(\sigma_3-\sigma_1)^2+C_3(\sigma_1-\sigma_2)^2+C_4\sigma_1+C_5\sigma_2+C_6\sigma_3+C_7\tau_{23}^2+C_8\tau_{31}^2+C_9\tau_{12}^2=1$$

For plane stress in the 1-2 plan ($\sigma_3=\tau_{23}=\tau_{31}=0$) and transverse isotropy in the 2-3 plane ($Z=Y$, $S_{31}=S_{12}$), the failure criterion simplifies to

$$\boxed{-\frac{\sigma_1^2}{X_cX_t}+\frac{\sigma_1\sigma_2}{X_cX_t}-\frac{\sigma_2^2}{Y_cY_t}+\frac{X_c+X_t}{X_cX_t}\sigma_1+\frac{Y_c+Y_t}{Y_cY_t}\sigma_2+\frac{\tau_{12}^2}{S_{12}^2}=1}$$

For equal strengths in tension and compression ($X_c=-X_t=-X$ and $Y_c=-Y_t=-Y$), the Hoffman failure criterion reduces to the Tsai-Hill criterion.

Both criteria are ellipsoids in $\sigma_1, \sigma_2, \tau_{12}$ space.

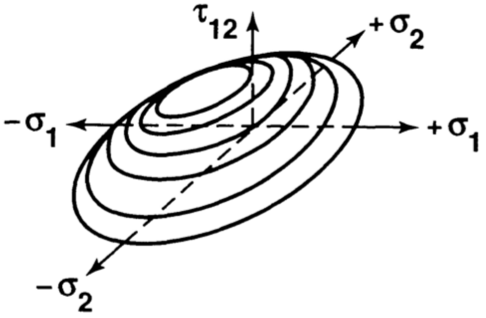



Figure 2-41 Hoffman Failure Surface

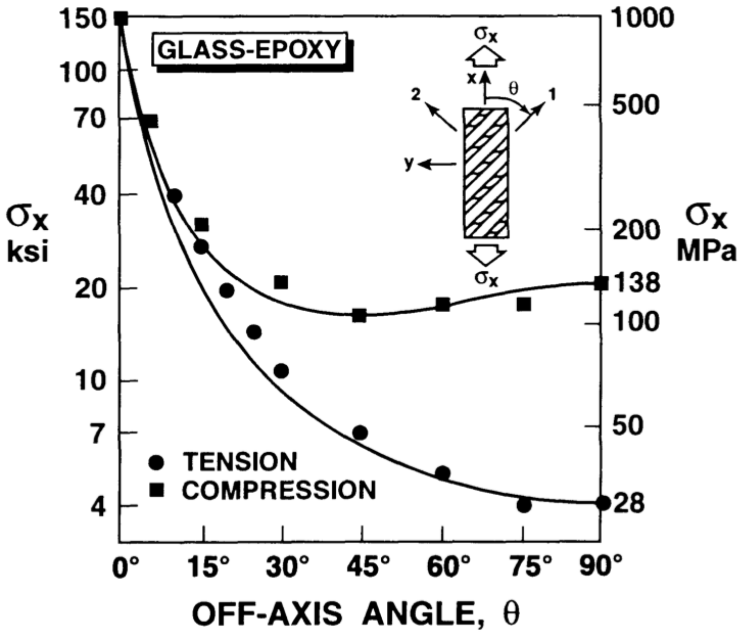



Figure 2-42 Hoffman Failure Criterion for Glass-Epoxy
*(Data from Tsai [2-21])*

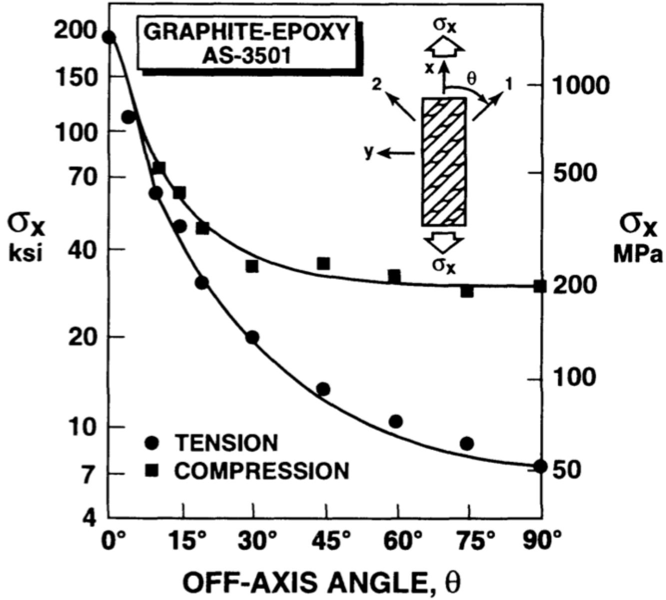



Figure 2-43 Hoffman Failure Criterion for Graphite-Epoxy
*(Data from Kim [2-24])*

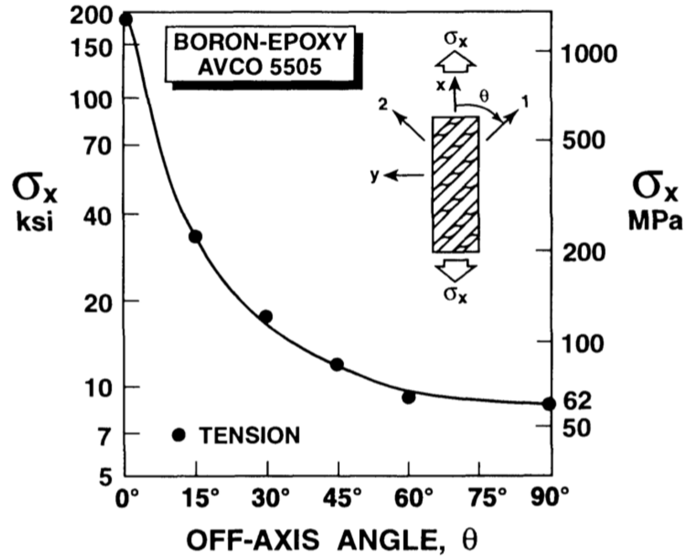



Figure 2-44 Hoffman Failure Criterion for Boron-Epoxy *(Data from Pipes and Cole [2-25])*

Attractive features of the Hoffman failure criterion are

(1) Interaction between failure modes is treated instead of separate criteria for failure like the maximum stress or maximum strain failure criteria.
(2) A single failure criterion is used in all quadrants of $\sigma_1$-$\sigma_2$ space instead of the segments in separate quadrants for the Tsai-Hill failure criterion because of different strengths in tension and compression.
(3) In design use, the Hoffman criterion is the simplest criterion of all criteria discussed.