Composites Lesson 11

بسم الله الرحمن الرحيم

SHEAR-EXTENSION COUPLING

BENDING-EXTENSION COUPLING

$$
\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}
$$

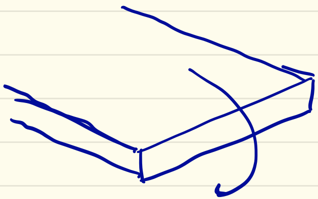
EXT_TWIST coupling

18 constants

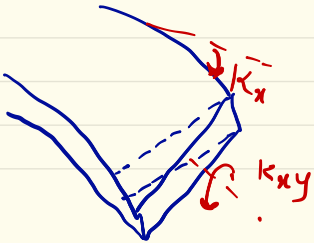
$$
\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}
$$

BENDING-EXTENSION COUPLING

BEND-TWIST COUPLING

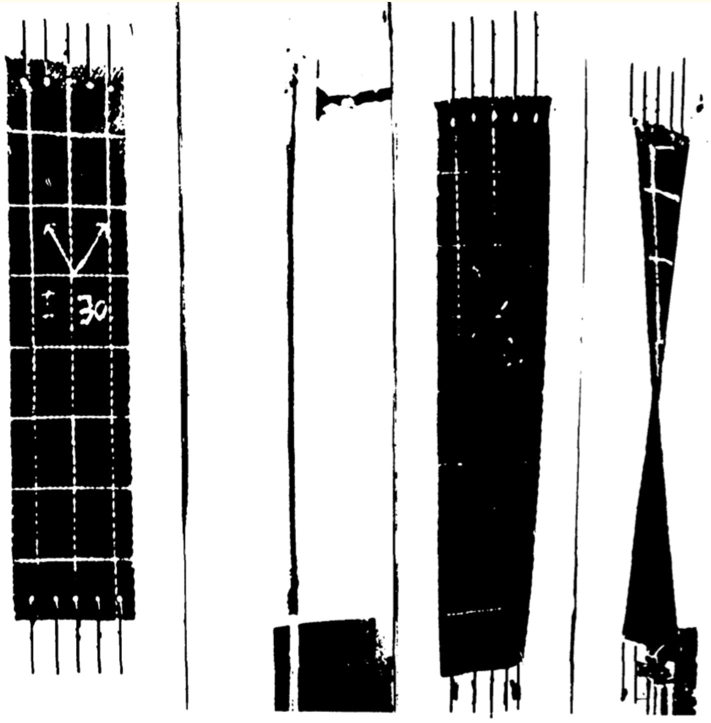


m_x

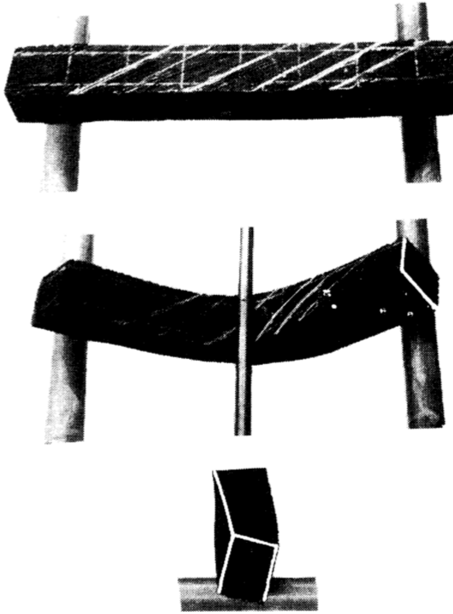


The effect of B_{16} & B_{26}

(extension - twist coupling)

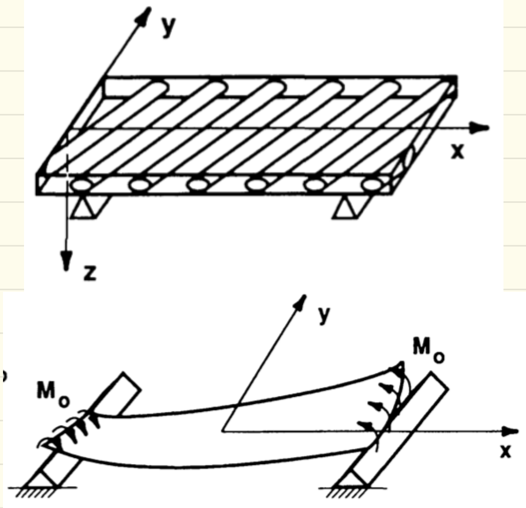


a Unloaded b Loaded with N_x

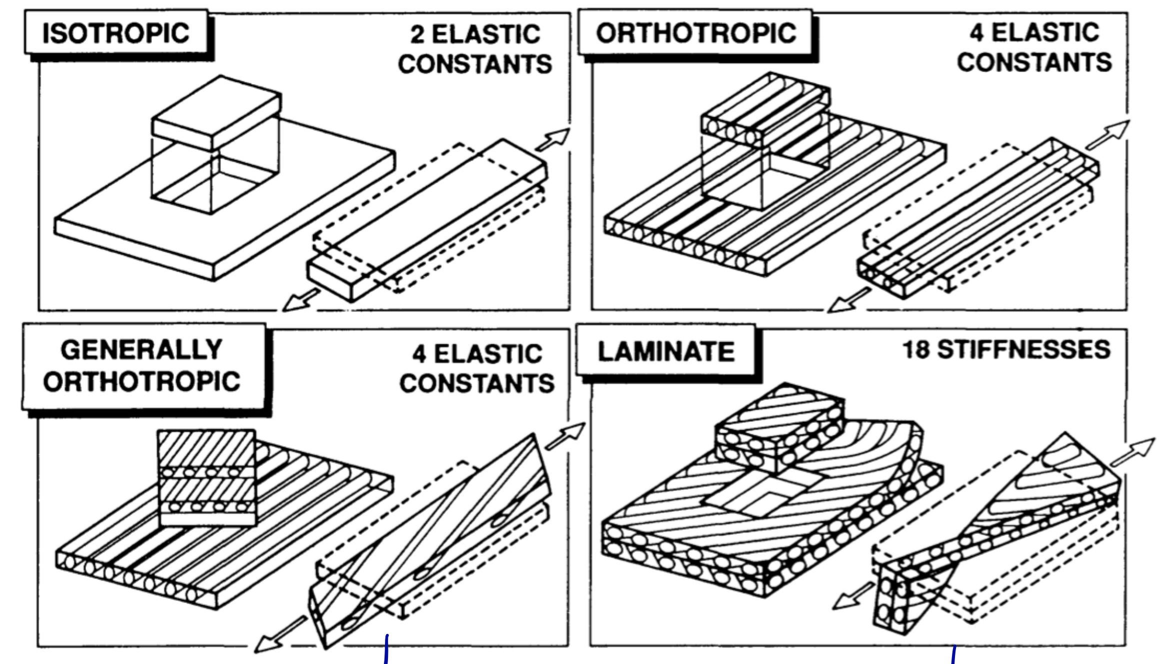


ANISOTROPIC

b Non-Aligned Orthotropy



The effect of D_{16}, D_{26} (bending-twist coupling)



shearing-extension Coupling

A_{16}, A_{26}

B_{16}, B_{26}

ext-twist coupling

4.4. Special Case of Laminate Stiffnesses

A. Single-Layered Configurations

we can prove that for symmetric layered lamiates (of course for single layered too) B_{ij} is zero.

A.1. Single Isotropic Layer

$B_{ij} = 0$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A & \nu A & 0 \\ \nu A & A & 0 \\ 0 & 0 & \frac{1-\nu}{2} A \end{bmatrix} \begin{bmatrix} \varepsilon_x^{\circ} \\ \varepsilon_y^{\circ} \\ \gamma_{xy}^{\circ} \end{bmatrix}$$

~ inplane-Strain

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D & \nu D & 0 \\ \nu D & D & 0 \\ 0 & 0 & \frac{1-\nu}{2} D \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

A.2 - Single Speciall orthotropic Layer $(\theta = 0°, 90°)$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

$$A_{11} = Q_{11}t \qquad D_{11} = \frac{Q_{11}t^3}{12}$$

$$A_{12} = Q_{12}t \qquad D_{12} = \frac{Q_{12}t^3}{12}$$

$$A_{22} = Q_{22}t \qquad B_{ij} = 0 \qquad D_{22} = \frac{Q_{22}t^3}{12}$$

$$A_{16} = 0 \qquad D_{16} = 0$$

$$A_{26} = 0 \qquad D_{26} = 0$$

$$A_{66} = Q_{66}t \qquad D_{66} = \frac{Q_{66}t^3}{12}$$

A.3 - Single Generally orthotropic Layer $(\theta = \alpha)$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

$$A_{ij} = \overline{Q}_{ij}\, t \qquad B_{ij} = 0 \qquad D_{ij} = \frac{\overline{Q}_{ij}\, t^3}{12}$$

A.4. Single Anisotropic Layer

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

$$A_{ij} = Q_{ij}\, t \qquad B_{ij} = 0 \qquad D_{ij} = \frac{Q_{ij}\, t^3}{12}$$

B. Symmetric Laminates

Laminates that are symmetric in both geometry and material properties about the middle surface is called symmetric laminates.

$$B_{ij} = 0$$

Symmetric laminates are commonly used unless special circumstances require an unsymmetric laminate.

B.1 - Symmetric laminates with multiple Isotropic Layers

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^{\circ} \\ \varepsilon_y^{\circ} \\ \gamma_{xy}^{\circ} \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

$A_{16} = A_{26} = 0$

$D_{16} = D_{26} = 0$

B.2 - symmetric Laminates with multiple spacially orthotropic Layers

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^{\circ} \\ \varepsilon_y^{\circ} \\ \gamma_{xy}^{\circ} \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

Symmetric Cross-ply Laminates ($\theta = 0°, 90°$)

$A_{16} = A_{26} = 0$

$D_{16} = D_{26} = 0$

B.3. Symmetric Laminates with multiple Generally orthotropic Layers

Symmetric Angle-ply Laminates ($\theta = +\alpha$ or $-\alpha$)

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

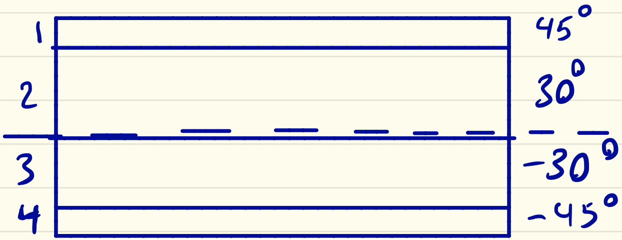
$A_{16}, A_{26}, D_{16}, D_{26}$ are small compared with the other A_{ij} & D_{ij}

C. Antisymmetric Laminates

Antisymmetry of a laminate requires

1- Symmetry about the middle surface of geometry

2. Some kind of a reversal or mirror image of the material properties ($[\bar{Q}]_k$)



$[B] \neq 0$, $A_{16} = A_{26} = 0$, $D_{16} = D_{26} = 0$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^\circ \\ \varepsilon_y^\circ \\ \gamma_{xy}^\circ \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^\circ \\ \varepsilon_y^\circ \\ \gamma_{xy}^\circ \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

C.1- Antisymmetric Cross-ply Laminates $(0^\circ, 90^\circ)$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} + \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & -B_{11} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & -B_{11} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

C.2. Antisymmetric Angle-Ply Laminates

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^\circ \\ \varepsilon_y^\circ \\ \gamma_{xy}^\circ \end{bmatrix} + \begin{bmatrix} 0 & 0 & B_{16} \\ 0 & 0 & B_{26} \\ B_{16} & B_{26} & 0 \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} 0 & 0 & B_{16} \\ 0 & 0 & B_{26} \\ B_{16} & B_{26} & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_x^\circ \\ \varepsilon_y^\circ \\ \gamma_{xy}^\circ \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

4.5. Some Notation

$[0/90/45]$ regular laminate (equal-thickness layers)

$[0^\circ_t/90^\circ_{2t}/45^\circ_{3t}]$ irregular laminate

$[0^\circ @ t / 90^\circ @ 2t / 45^\circ @ 3t]$

$[0^\circ/90^\circ/45^\circ]_S \equiv [0^\circ/90^\circ/45^\circ/45^\circ/90^\circ/0^\circ]$

$[0^\circ/0^\circ/0^\circ/90^\circ/90^\circ] \equiv [0^\circ_3/90^\circ_2]$

$[0^\circ/90^\circ/45^\circ/0^\circ/90^\circ/45^\circ] \equiv [0^\circ/90^\circ/45^\circ]_2$

$[0^\circ/90^\circ/0^\circ] \equiv [0^\circ/\overline{90}^\circ]_S$

Quasi-Isotropic Laminates ≡ equal extensional stiffnesses in all in-plane directions of the laminar

$A_{11} = A_{22}$, $A_{12} = \nu A_{11}$, $A_{16} = A_{26} = 0$

example: $[-60^\circ/0^\circ/60^\circ]$ or $[0^\circ/-45^\circ/45^\circ/90^\circ]$