Composites Lesson 10 بسم الله الرحمن الرحيم

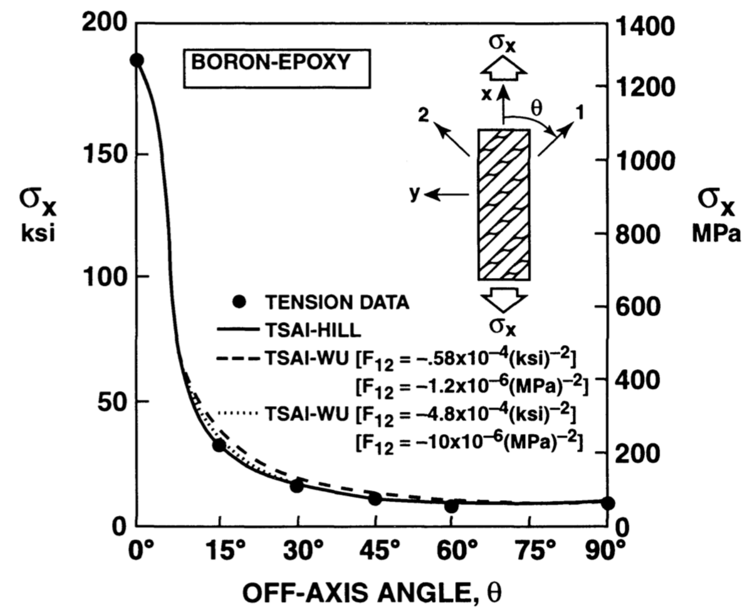
⑤ Tsai-Wu Tensor Failure Criterion

$$F_{11}\sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + F_1\sigma_1 + F_2\sigma_2 = 1$$

$$F_{11} = -\frac{1}{X_t X_c}, \quad F_{22} = -\frac{1}{Y_t Y_c}, \quad F_{66} = \frac{1}{S^2}$$

$$F_1 = \frac{1}{X_t} + \frac{1}{X_c}, \quad F_2 = \frac{1}{Y_t} + \frac{1}{Y_c}$$

If $\tau_{12} = 0$ then $F_{12} = -\frac{1}{2}(F_{11}F_{22})^{1/2}$ can be considered otherwise its value must be obtained experimentally.



BORON-EPOXY
TENSION DATA
TSAI-HILL
TSAI-WU [F12 = −.58x10−4(ksi)−2]
[F12 = −1.2x10−6(MPa)−2]
TSAI-WU [F12 = −4.8x10−4(ksi)−2]
[F12 = −10x10−6(MPa)−2]
σx
ksi
σx
MPa
OFF-AXIS ANGLE, θ
0° 15° 30° 45° 60° 75° 90°
0 50 100 150 200
0 200 400 600 800 1000 1200 1400
x y 1 2 θ

The Tsai-Wu *failure criterion* has several important characteristics:

(1) Increased curve-fitting capability over the Tsai-Hill and Hoffman criteria because of an additional term in the equation.
(2) The additional term, F_{12}, can be determined only with an expensive and difficult-to-perform biaxial test.
(3) Graphical interpretations of the results are facilitated by the tensor formulation.

⑥ Puck Failue Criterion

$$
\begin{cases}
\dfrac{\sigma_1}{X} \leq 1 \\[2ex]
\left(\dfrac{\sigma_2}{Y}\right)^2 + \left(\dfrac{\tau_{12}}{S}\right)^2 \leq 1
\end{cases}
$$

⑦ Puppo-Evensen Failure Criterion

$$
\left(\frac{\sigma_1}{X}\right)^2 - \phi \frac{X}{Y} \frac{\sigma_1}{X} \frac{\sigma_2}{Y} + \phi \left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2 \leq 1
$$

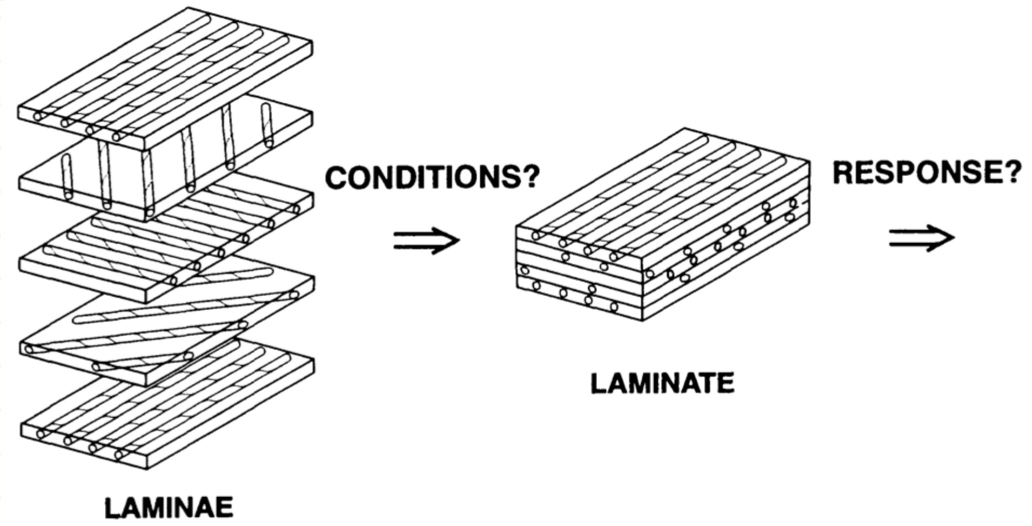
$$
\phi \left(\frac{\sigma_1}{X}\right)^2 - \phi \frac{X}{Y} \frac{\sigma_1}{X} \frac{\sigma_2}{Y} + \left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2 \geq 1
$$

$$
\phi = \frac{3S^2}{X_t Y_t}
$$

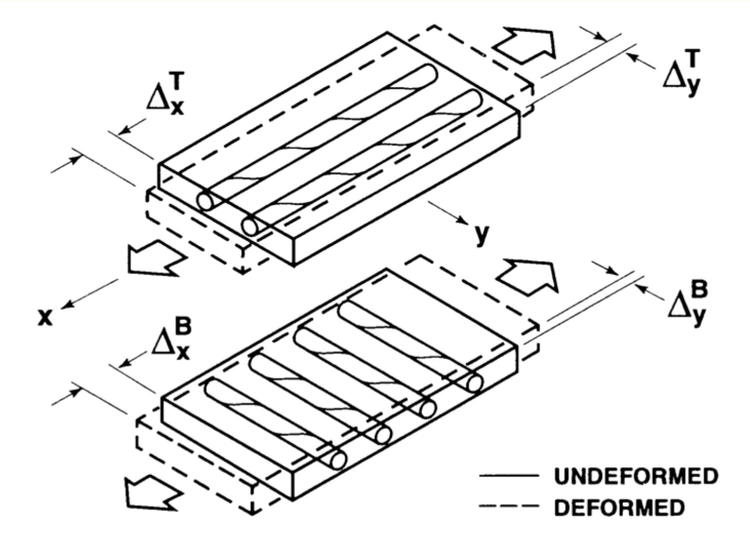
Chapter 4. Macromechanical Behavior of a Laminate

4.1. Introduction

A laminate is two or more laminae bonded together to act as an integral structural element.



The reason laminae are combined to create a laminate is to achieve the largest possible bending stiffness for the material used.

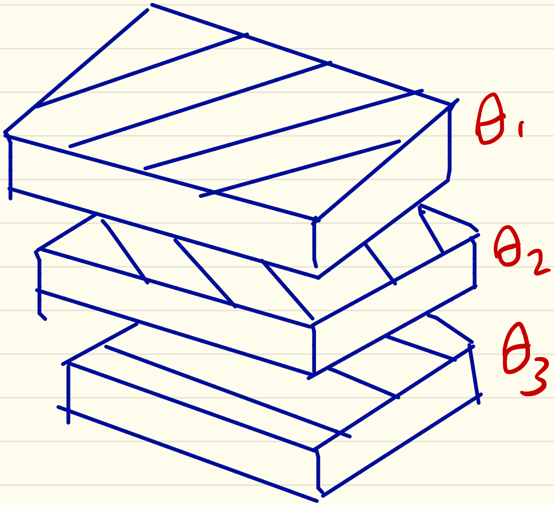


4.2. Strain and stress Variation in a Laminate

$\{\sigma\} = [\bar{Q}]_1 \{\varepsilon\}$

$\{\sigma\} = [\bar{Q}]_2 \{\varepsilon\}$

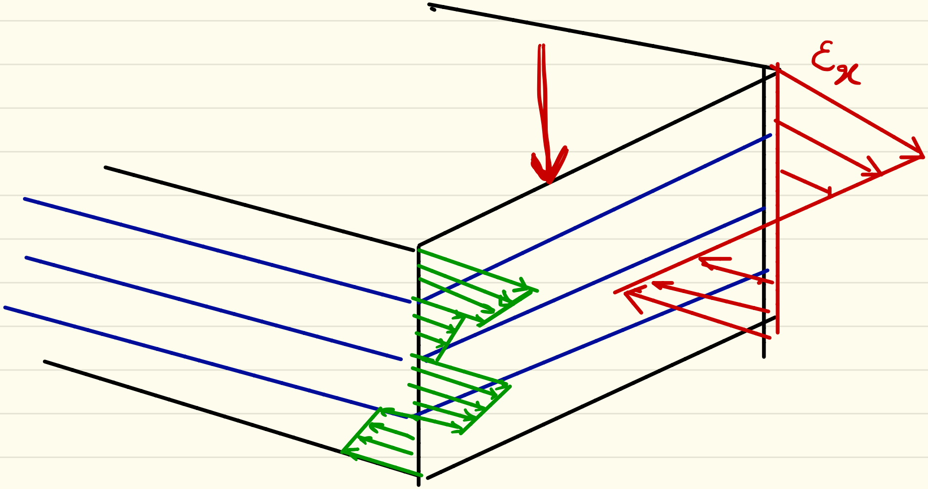
$\{\sigma\} = [\bar{Q}]_3 \{\varepsilon\}$

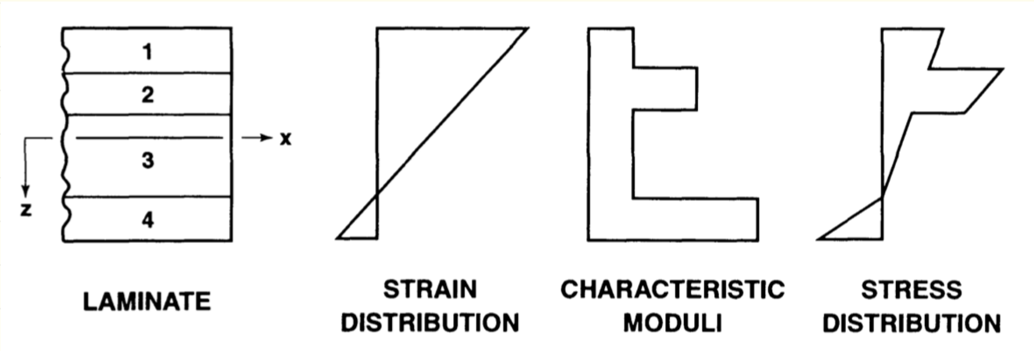


$[\theta_1, \theta_2, \theta_3]$

$\theta_1 / \theta_2 / \theta_3$

$[90, 30, 60]_s$


ε_x


1
2
3
4
x
z
LAMINATE
STRAIN DISTRIBUTION
CHARACTERISTIC MODULI
STRESS DISTRIBUTION

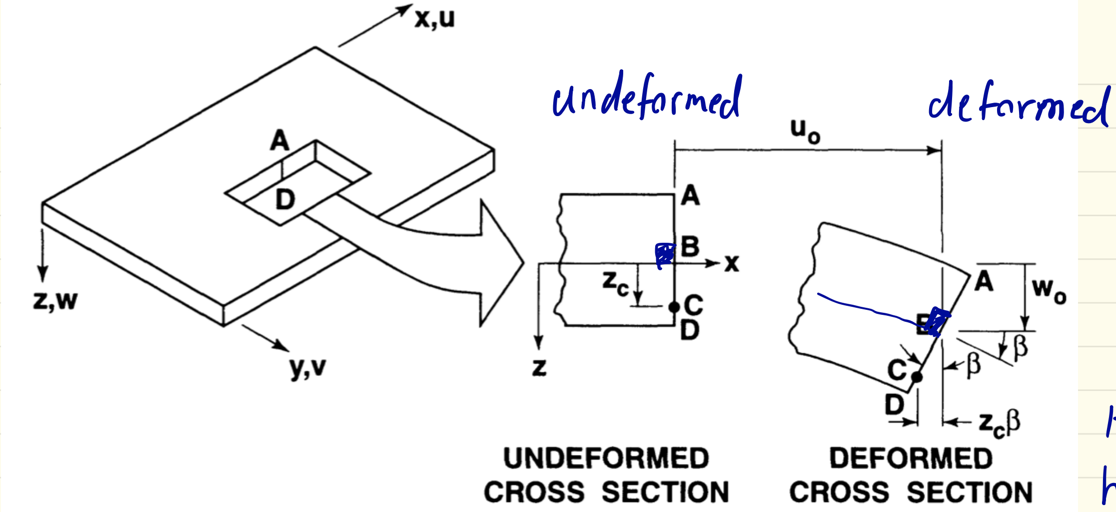


Figure 4-4 Geometry of Deformation in the x-z Plane

Kirchhoff hypothesis

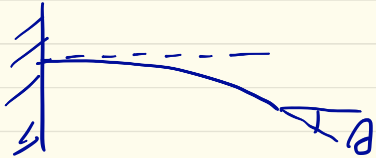
Accordingly, if the laminate is thin, a line originally straight and perpendicular to the middle surface of the laminate, i.e., a normal to the middle surface, is assumed to remain straight and perpendicular to the middle surface when the laminate is deformed, e.g., bent, extended,

$\Rightarrow \gamma_{xz} = \gamma_{yz} = 0$

Assumption: ε_z can be ignored ($\varepsilon_z = 0$)

The symbol 'nought' (0) is used to designate middle-surface values of a variable.

$$u_c = u_0 - z_c \beta \quad , \beta = \frac{\partial w_0}{\partial x}$$



$\theta = \frac{dw}{dx}$

$$u(x, y, z) = u_0(x, y) - z \frac{\partial w_0}{\partial x} \quad , \quad v = v_0 - z \frac{\partial w_0}{\partial y}$$

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w_0}{\partial x^2}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} - z \frac{\partial^2 w_0}{\partial x^2}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right) - 2 z \frac{\partial^2 w_0}{\partial x \partial y}$$

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix}$$

middle-surface strains

$$\begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}$$

middle-surface curvatures

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$

linear variation of strain through the laminate thickness

$$\boxed{\{\varepsilon\} = \{\varepsilon\}^0 + z\{K\}}$$

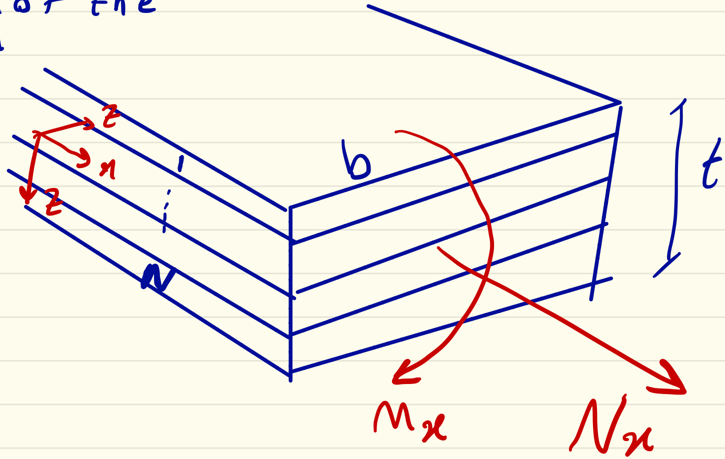
$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left[\begin{bmatrix} \varepsilon_x^\circ \\ \varepsilon_y^\circ \\ \gamma_{xy}^\circ \end{bmatrix} + z \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \right]$$

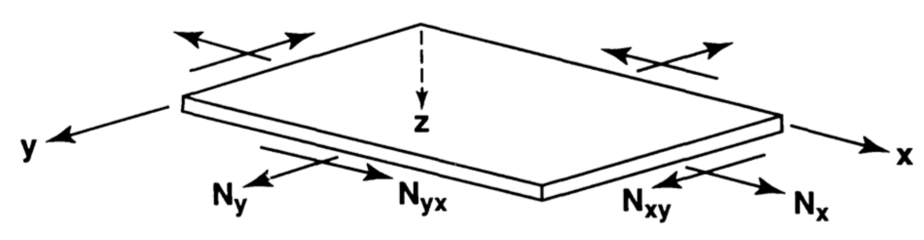
4.3. Resultant Laminate Forces and moments

$$N_x = \int_{-t/2}^{+t/2} \sigma_x \, dz$$

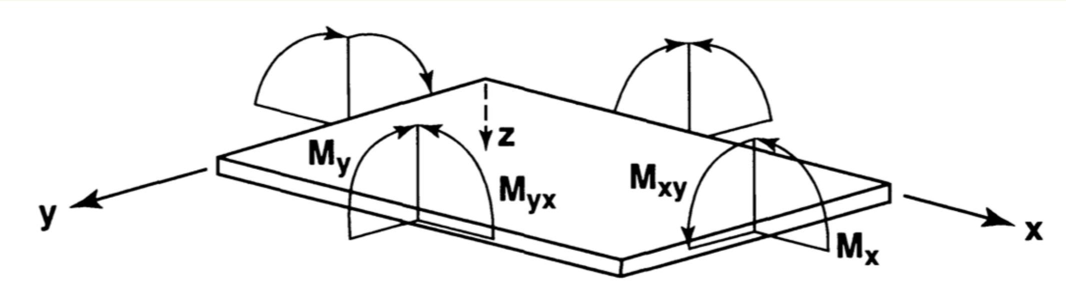
Per unit withof the cross section

$$M_x = \int_{-t/2}^{+t/2} z \sigma_x \, dz$$





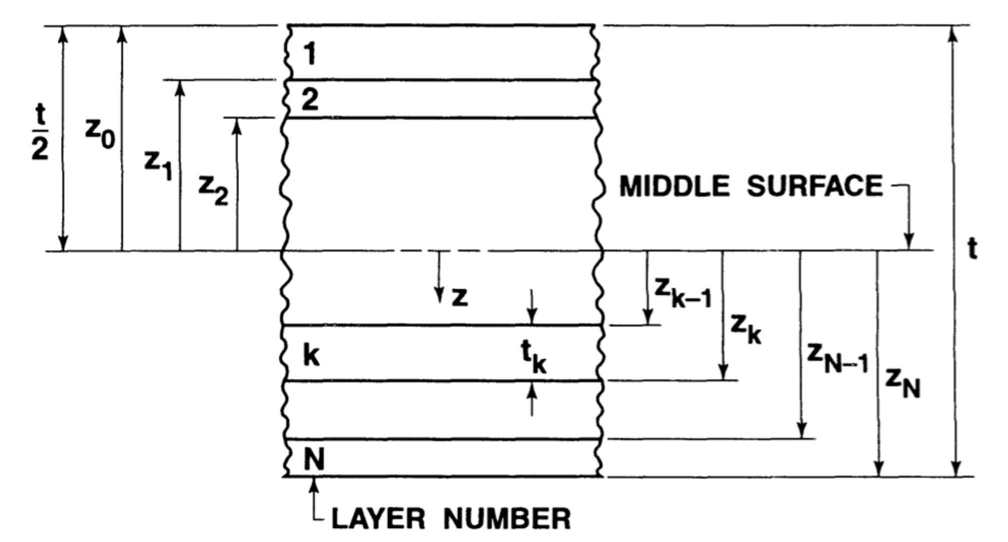
z
y
x
N_y
N_{yx}
N_{xy}
N_x



z
y
x
M_y
M_{yx}
M_{xy}
M_x

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-t/2}^{+t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-t/2}^{+t/2} z \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} z \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz$$



$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{N} [\bar{Q}]_k \int_{z_{k-1}}^{z_k} \left(\{\varepsilon\}^{\circ} + z\{k\} \right) dz$$

$$= \underbrace{\left(\sum_{k=1}^{N} [\bar{Q}]_k (z_k - z_{k-1}) \right)}_{[A]} \{\varepsilon\}^{\circ} + \underbrace{\left(\sum_{k=1}^{N} [\bar{Q}]_k \tfrac{1}{2} (z_k^2 - z_{k-1}^2) \right)}_{[B]} \{k\}$$

$$\{N\} = [A]\{\varepsilon\}^{\circ} + [B]\{k\}$$

$$\{M\} = [B]\{\varepsilon\}^{\circ} + [D]\{k\}$$

$$\boxed{\begin{aligned} [A] &= \sum_{k=1}^{N} [\bar{Q}]_k (z_k - z_{k-1}) \\ [B] &= \sum_{k=1}^{N} [\bar{Q}]_k \tfrac{1}{2} (z_k^2 - z_{k-1}^2) \\ [D] &= \sum_{k=1}^{N} [\bar{Q}]_k \tfrac{1}{3} (z_k^3 - z_{k-1}^3) \end{aligned}}$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^\circ \\ \varepsilon_y^\circ \\ \gamma_{xy}^\circ \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^\circ \\ \varepsilon_y^\circ \\ \gamma_{xy}^\circ \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

A: extensional stiffnesses

B: bending-extension coupling stiffnesses

D: bending stiffnesses

$$\boxed{\left\{ \begin{array}{c} N \\ \hline M \end{array} \right\} = \left[\begin{array}{c|c} A & B \\ \hline B & D \end{array} \right] \left\{ \begin{array}{c} \varepsilon^\circ \\ \hline \kappa \end{array} \right\}}$$