Composites Lesson 8 بسم الله الرحمن الرحيم.

3.8 - Engineering constants for a Lamina of Arbitrary Orientation

$E_x, E_y, \nu_{xy}, G_{xy}, \eta_{xy,x}, \eta_{xy,y}$

$E_x = \frac{\varepsilon_x}{\sigma_x}$ for $\sigma_1 = \sigma$ and all other stresses are zero

$$\frac{1}{E_X} = \frac{1}{E_1}cos^4\theta + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)sin^2\theta cos^2\theta + \frac{1}{E_2}sin^4\theta$$

$$\frac{1}{E_Y} = \frac{1}{E_1}sin^4\theta + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)sin^2\theta cos^2\theta + \frac{1}{E_2}cos^4\theta$$

$$\nu_{XY} = E_X\left[\frac{\nu_{12}}{E_1}(sin^4\theta + cos^4\theta) - \left(\frac{1}{E_1} + \frac{1}{E_2} - \frac{1}{G_{12}}\right)sin^2\theta cos^2\theta\right]$$

$$\frac{1}{G_{XY}} = 2\left(\frac{2}{E_1} + \frac{2}{E_2} + \frac{4\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)sin^2\theta cos^2\theta + \frac{1}{G_{12}}(sin^4\theta + cos^4\theta)$$

$$\eta_{XY,X} = E_X\left[\left(\frac{2}{E_1} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)sin\theta cos^3\theta - \left(\frac{2}{E_2} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)sin^3\theta cos\theta\right]$$

$$\eta_{XY,Y} = E_Y\left[\left(\frac{2}{E_1} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)sin^3\theta cos\theta - \left(\frac{2}{E_2} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)sin\theta cos^3\theta\right]$$

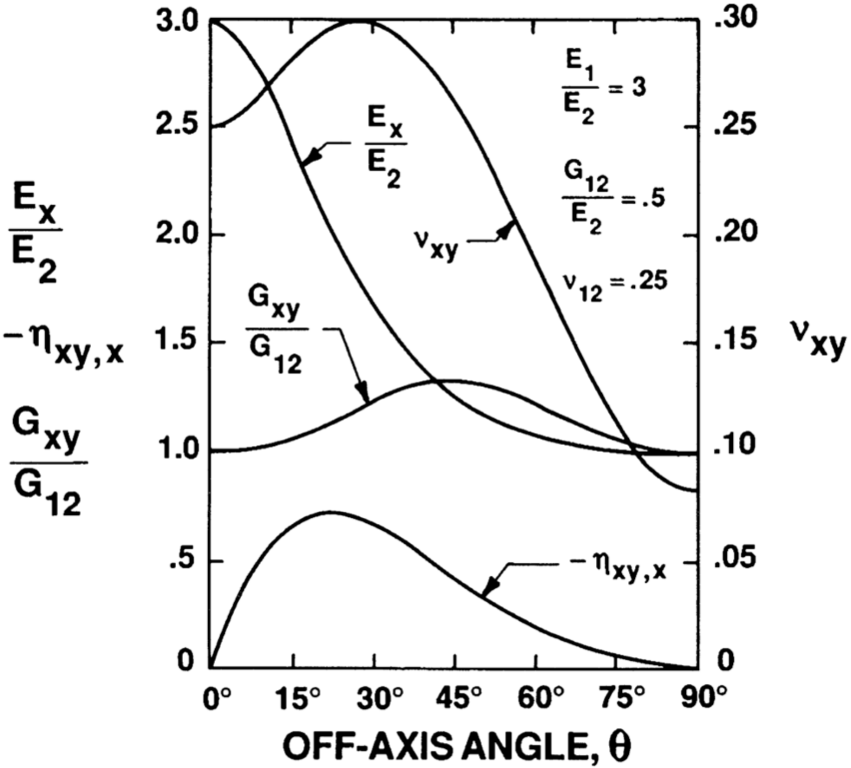


Figure 2-11 Normalized Moduli for Glass-Epoxy

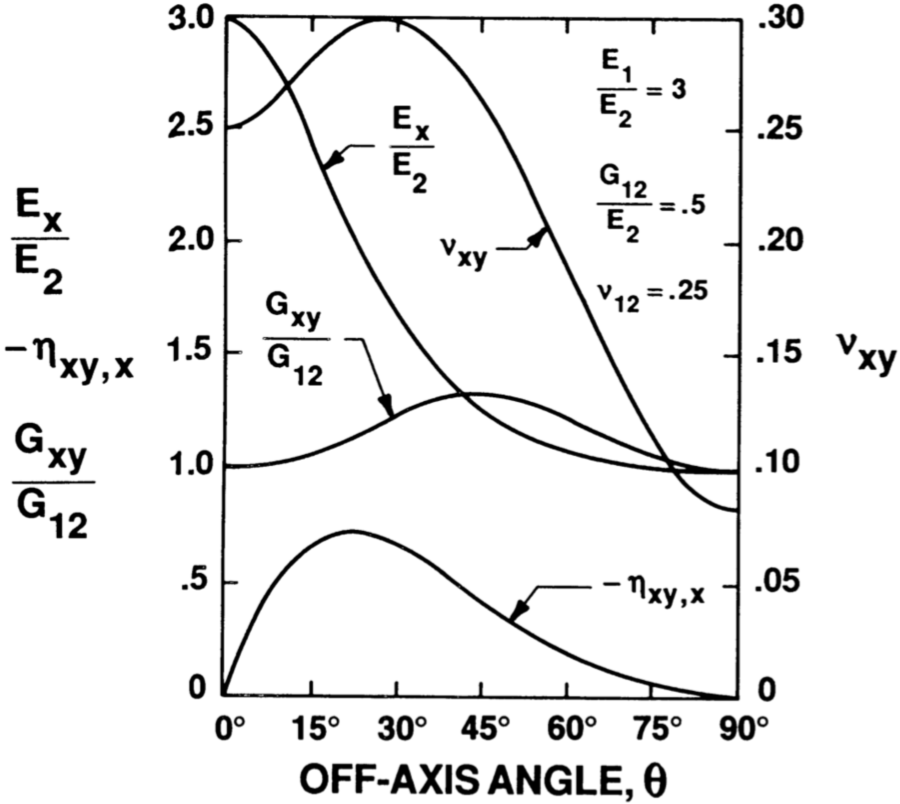
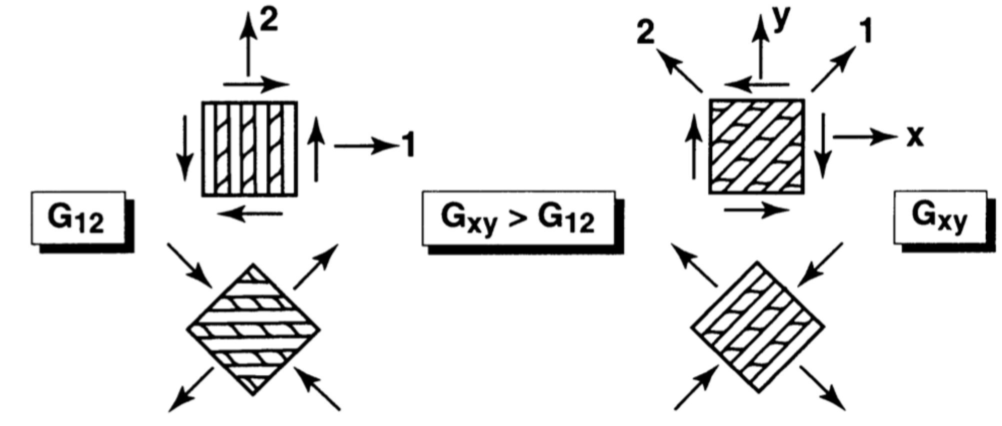
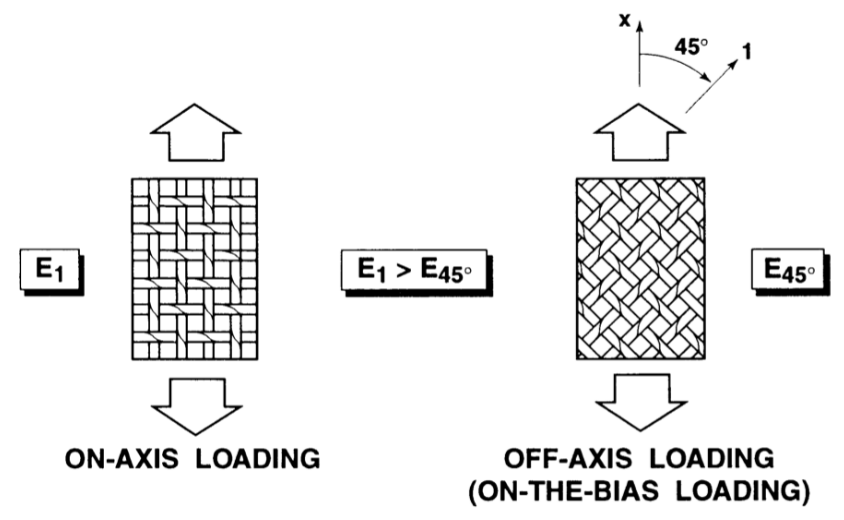


Figure 2-12 Normalized Moduli for Boron-Epoxy

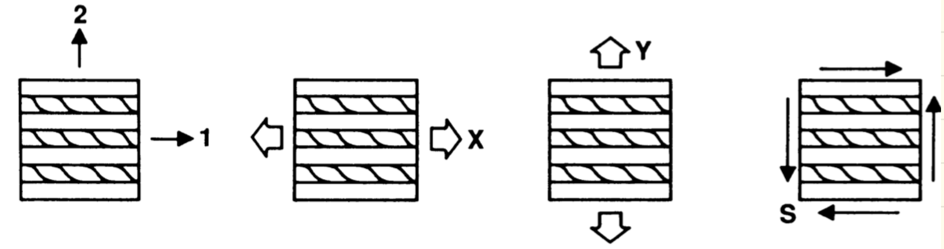
The extremum (largest and smallest) material properties do not necessarily occur in principal material coordinates. G_{xy} exceeds G_{12}, and $E_{45°}$ is less than E_1 for composite materials that have a fiber modulus much greater than the matrix modulus.





x
45°
1
E1
E1 > E45°
E45°
ON-AXIS LOADING
OFF-AXIS LOADING
(ON-THE-BIAS LOADING)

3.9 - Strength of an orthotropic Lamina



X_t = axial or longitudinal strength in tension

X_c = axial or longitudinal strength in compression

Y_t = transverse strength in tension

Y_c = transverse strength in compression

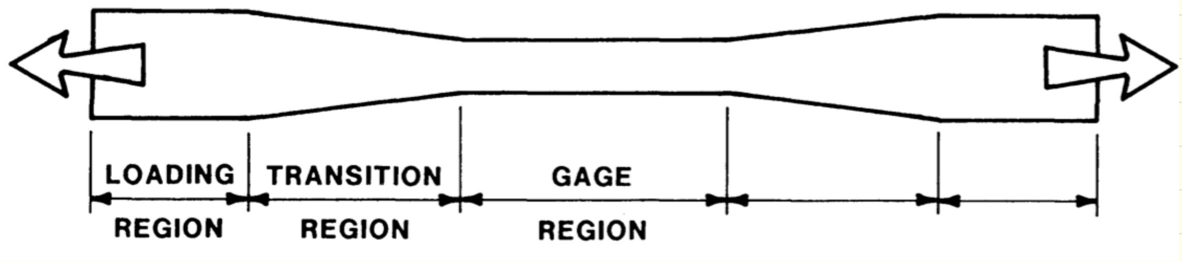
S = shear strength

3.9.1. Experimental Determination of Strength and Stiffness

A key point in the experimental tests is the imposition of a uniform stress state in the specimen. Because of coupling between shear and normal stresses and strains in composite materials, it is not so easy as for isotropic materials.

(1) The highest stress must occur in the gage section (region of smallest cross-sectional area) so that failure occurs in the gage section.
(2) A uniform stress field must exist over the entire gage-section volume to eliminate volume-based statistical failure effects (e.g., a realistic distribution of ordinary defects must exist for the test to be representative of the actual material).
(3) Unwanted 'other' stresses must be eliminated from the gage section (e.g., eliminate bending stresses induced by load-application mechanisms such as misalignment of loading grips).
(4) Alternatively to (3), *account for* certain end and edge effects (e.g., shear-extension coupling) in the data-reduction process.
(5) The specimen material and the test procedure must be representative of the intended application from the standpoint of
    (a) fabrication (a tape-laid specimen does not in any way represent a filament-wound structure!)
    (b) size effects (the characteristic dimensions of the specimen, e.g., thickness, cannot approach any characteristic material dimension such as void size, fiber diameter, etc.)
    (c) environment (the loading rate, moisture content, and temperature of the specimen must be similar to, if not identical with, the actual structural application)

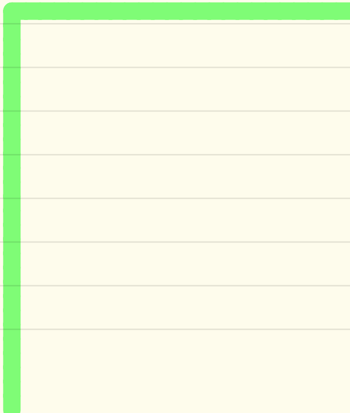
tension specimen



load is applied to specimen

without stress concentration

uniform and maximum load



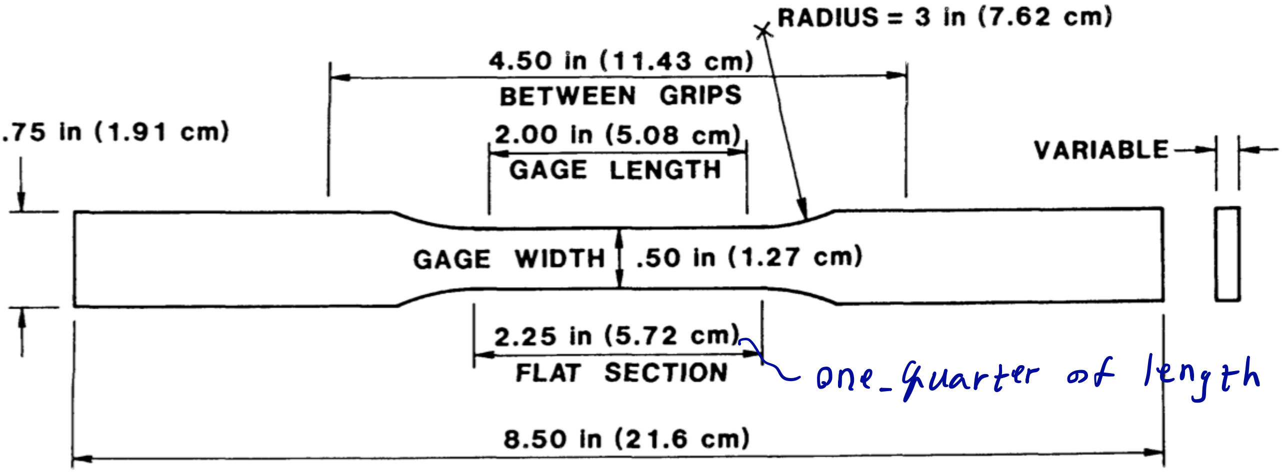


Figure 2-21 ASTM D 638 Tension Specimen

Here, the typical failure occurs in the transition region but the filure strength in this test is understimate of the real strength.

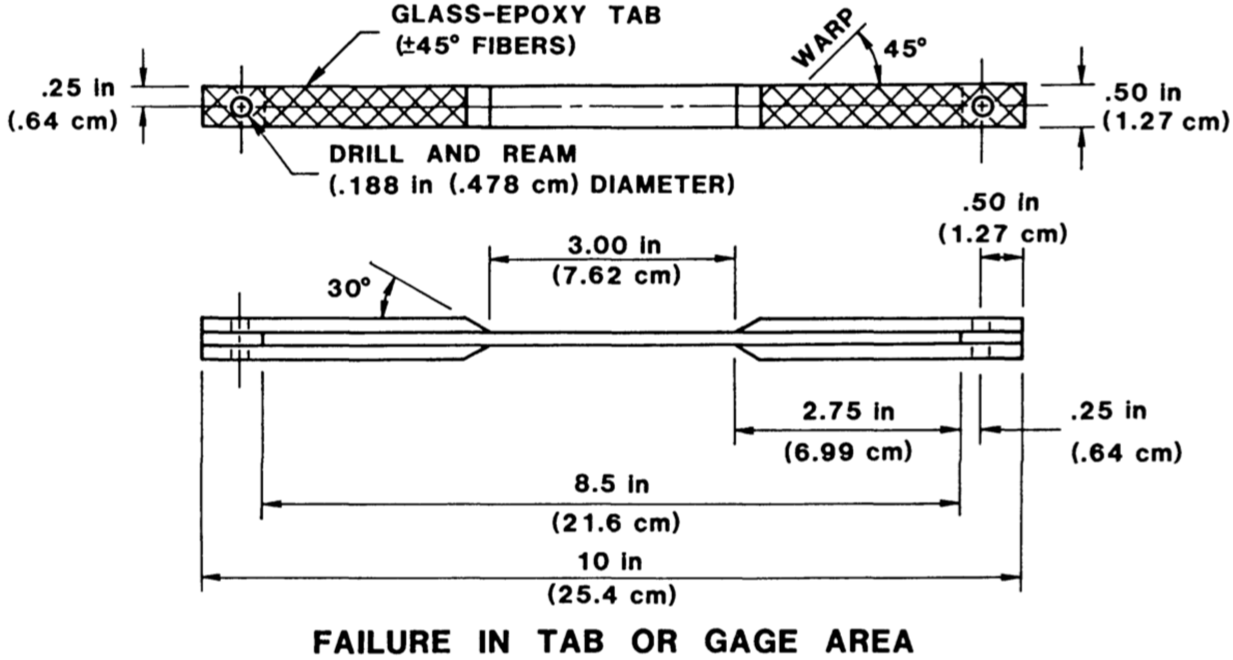


Figure 2-22 Straight-Sided Tension Specimen

Failures typically occur either in the bonded tabs or in the gage section.

↳ repeat

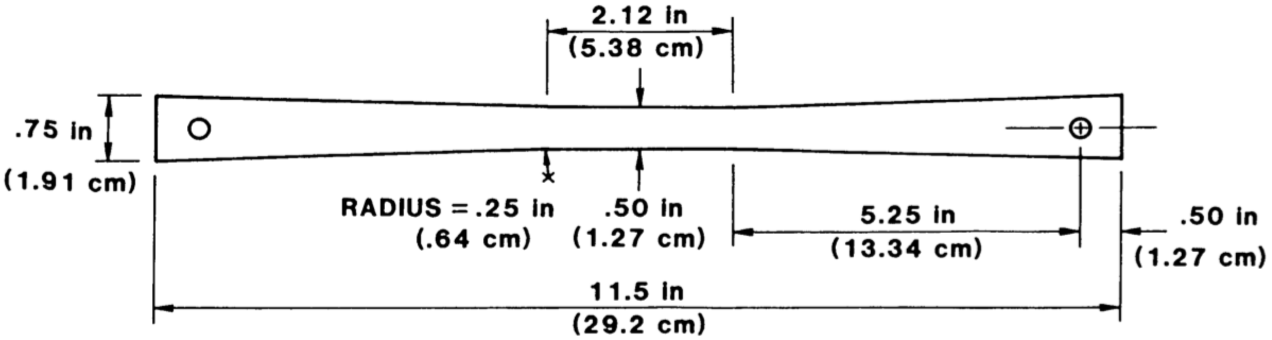


Figure 2-23 Bow-Tie Tension Specimen

It's longer than the previous two. Failure consistently occurs in the gage region (because it creats a very gradual transition region).

Under compression loading, the long flexible tension specimens would simply buckle. Thus, lateral support to prevent buckling is necessary as shown in the compression test fixture with side-support plates in Figure 2-24. There, the specimen is essentially as long as the fixture is tall, and only a small portion of the specimen can be seen where it is not supported.

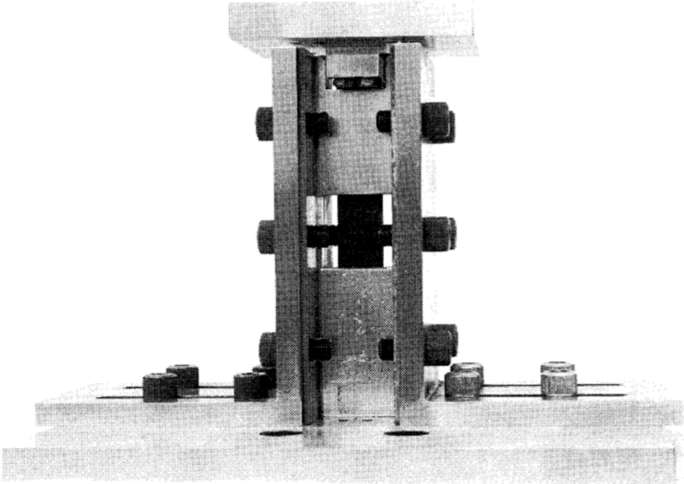
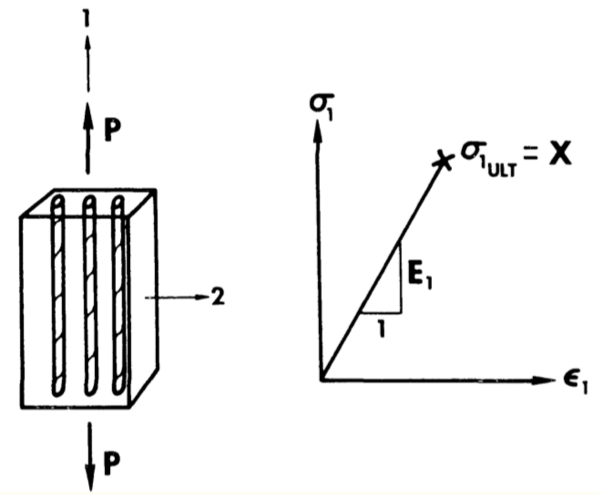


Figure 2-24 Compression Test Fixture

Test-1

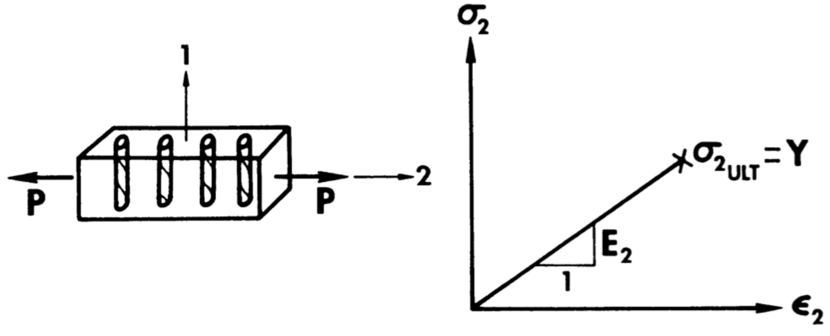


$$\left.\begin{array}{l}\sigma_1 = \dfrac{P}{A} \\ \varepsilon_1 = \dfrac{\Delta}{\ell}\end{array}\right\} \Rightarrow E_1 = \frac{\sigma_1}{\varepsilon_1}$$

$$X_t, X_c = \frac{P_{ult}}{A}$$

$$\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1}$$

test_2



$$\sigma_2 = \frac{P}{A}, \quad \varepsilon_2 = \frac{\Delta}{l} \quad \Rightarrow \quad E_2 = \frac{\sigma_2}{\varepsilon_2}$$

$$Y_t, Y_c = \frac{P_{ult}}{A}$$

$$\nu_{21} = -\frac{\varepsilon_1}{\varepsilon_2}$$

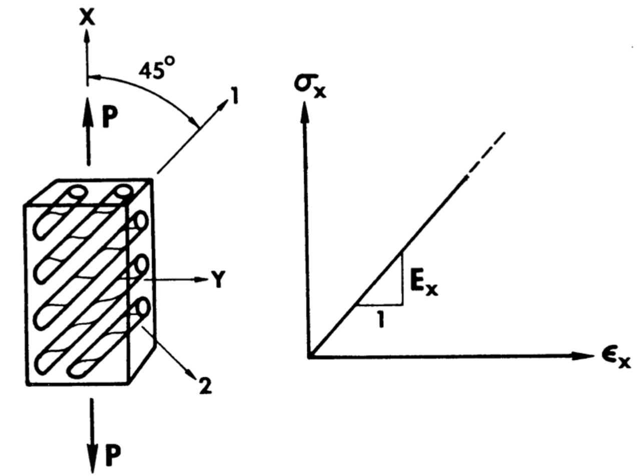
$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}$$

else one of three possibilities exists:

(1) The data were measured incorrectly
(2) The calculations were performed incorrectly
(3) The material cannot be described with linear elastic stress-strain relations

Test_3

To determine the reaining properties G_{12} and S, consider uniaxial tension load on a flat 45° direction piece of lamina.



$$\left.\begin{aligned} \sigma_x &= \frac{P}{A} \\ \varepsilon_x &= \frac{\Delta}{\ell} \end{aligned}\right\} \Rightarrow E_x = \frac{\sigma_x}{\varepsilon_x}$$

we showed that

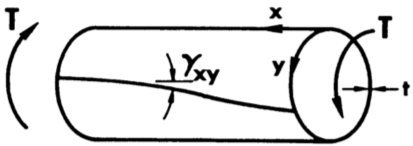
$$\frac{1}{E_x} = \frac{1}{4}\left[\frac{1}{E_1} - \frac{2\nu_{12}}{E_1} + \frac{1}{G_{12}} + \frac{1}{E_2}\right]$$

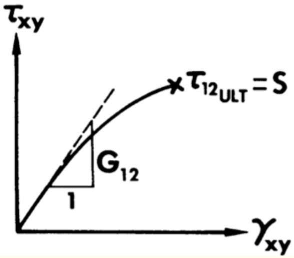
$$\rightarrow \quad G_{12} = \frac{1}{\frac{4}{E_x} - \frac{1}{E_1} - \frac{1}{E_2} + \frac{2\nu_{12}}{E_1}}$$

This test cannot be relied upon to determine S.

Test_4

Torsion-tube test.





$$\tau_{12} = \frac{T}{2\pi r^2 t}$$

$$S = \tau_{12_{ult}} = \frac{T_{ult}}{2\pi r^2 t}$$

$$G_{12} = \frac{\tau_{12}}{\gamma_{12}}$$

Property	Unidirectionally Reinforced Composite Material			
	Glass-Epoxy	Boron-Epoxy	Graphite-Epoxy	Kevlar®-Epoxy
E_1	54 GPa	207 GPa	207 GPa	76 GPa
E_2	18 GPa	21 GPa	5 GPa	5.5 Gpa
ν_{12}	.25	.3	.25	.34
G_{12}	9 GPa	7 GPa	2.6 GPa	2.1 GPa
X_t	1035 MPa	1380 MPa	1035 MPa	1380 MPa
Y_t	28 MPa	83 MPa	41 MPa	28 MPa
S	41 MPa	124 MPa	69 MPa	44 MPa
X_c	1035 MPa	2760 MPa	689 MPa	276 MPa
Y_c	138 MPa	276 MPa	117 MPa	138 MPa

3.9_2_ Biaxial Strength Criteria for an Orthotropic Lamina