Composites Lesson 7 بسم الله الرحمن الرحيم

3.4. Restrictions on Engineering Constants

3.4.1 Isotropic Materials

$$G = \frac{E}{2(1+\nu)}$$

$$\sigma = E\varepsilon$$

$$E, G > 0 \Longrightarrow -1 < \nu$$

Volumetric strain

$$\varepsilon = \frac{\sigma}{E}$$

$$\theta = \frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{P}{K}$$

P → Hydrostatic Pressure

K → bulk modulus

$$K = \frac{E}{3(1-2\nu)}$$

$$K > 0 \Longrightarrow \nu < \frac{1}{2} \Rightarrow -1 < \nu < \frac{1}{2}$$

3.4-2- orthotropic Materials

we showed that

$$S_{ij} = S_{ji} \longrightarrow \frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j}$$

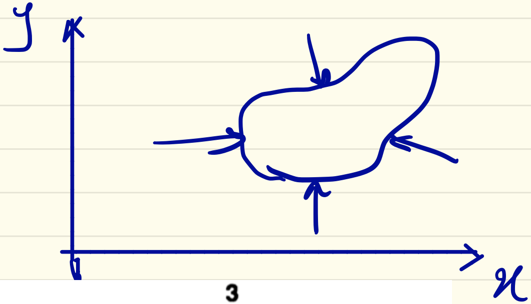
If only one normal stress is applied at a time, the corresponding Strain is determined by the diagonal elements of the compliance matrix. Thus, Those elements must be positive.

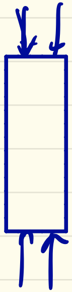
$$S_{11}, S_{22}, S_{33}, S_{44}, S_{55}, S_{66} > 0$$

$$\Rightarrow E_1, E_2, E_3, G_{12}, G_{13}, G_{23} > 0$$

we can write some other equations which are not so usefull for remembering them.

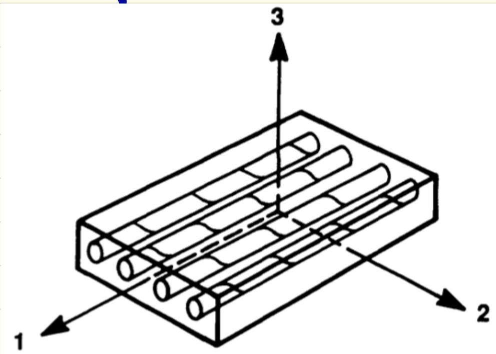
3.5_ Stress_Strain Relations for Plane Stress orthotropic materials





only inplane_load

$\sigma_3 = 0 = \sigma_{31} = \sigma_{32}$



Principal material coordinates

(Principal directions of orthotropy)

The strain-stress relation for orthotropic materials reduce to

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

$$S_{11} = \frac{1}{E_1}, \quad S_{22} = \frac{1}{E_2}, \quad S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2}, \quad S_{66} = \frac{1}{G_{12}}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

$$\varepsilon_3 = S_{13}\sigma_1 + S_{23}\sigma_2, \quad \gamma_{23} = 0, \quad \gamma_{31} = 0$$

$$S_{13} = -\frac{\nu_{13}}{E_1} = -\frac{\nu_{31}}{E_3}, \quad S_{23} = -\frac{\nu_{23}}{E_2} = -\frac{\nu_{32}}{E_3}$$

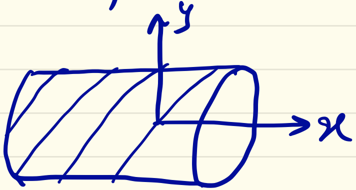
Mustlly composite materials are used as thin shells, therefore ε_3 is negligible.

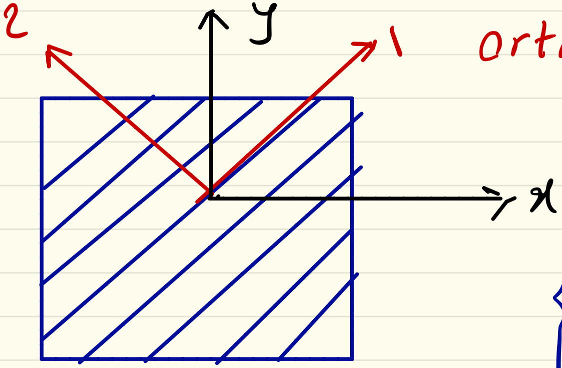
For plane stress on isotropic materials, we have

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

3.6_ Stress_Strain Relations for a Lamina of Arbitrary Direction

Sometimes, the geometricall coordinate directions of the body do not concide with principal material coordinates (For example wound fiber_reinforced circular cylindrical shells)

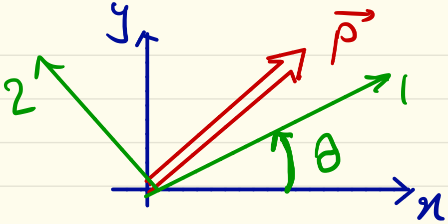




orthotropy Coordinates

Geometrical coordinates

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \left[\bar{Q}\right] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$



$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = [T(\theta)] \begin{Bmatrix} P_x \\ P_y \\ P_z \end{Bmatrix}$$

Transformation or rotation matrix

$$[T(\theta)] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

θ: off-axis angle = angle from the x-axis to 1-axis

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = [T(\theta)] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

$$T(-\theta) = T^{-1}(\theta)$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{\gamma_{12}}{2} \end{Bmatrix} = [T(\theta)] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{\gamma_{xy}}{2} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [R] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{\gamma_{xy}}{2} \end{Bmatrix} , \quad [R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

$$\to [Q] \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \to [R] \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{\gamma_{12}}{2} \end{Bmatrix} \to [T] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{\gamma_{xy}}{2} \end{Bmatrix} \to [R]^{-1} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\{\sigma_x\} = \overbrace{[T]^{-1}[Q]\underbrace{[R][T][R]^{-1}}_{[R][T][R]^{-1} = [T]^{-T}}}^{[\bar{Q}]} \{\varepsilon_x\}$$

$$\boxed{[\bar{Q}] = [T]^{-1}[Q][T]^{-T}}$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})sin^2\theta cos^2\theta + Q_{12}(sin^4\theta + cos^4\theta)$$

$$\bar{Q}_{22} = Q_{11}sin^4\theta + 2(Q_{12} + 2Q_{66})sin^2\theta cos^2\theta + Q_{22}cos^4\theta$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})\sin\theta\, cos^3\theta + (Q_{12} - Q_{22} + 2Q_{66})sin^3\theta \cos\theta$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})sin^3\theta \cos\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin\theta\, cos^3\theta$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})sin^2\theta cos^2\theta + Q_{66}(sin^4\theta + cos^4\theta)$$

$$\overline{S}_{12} = (S_{11} + S_{22} - S_{66})sin^2\theta cos^2\theta + S_{12}(sin^4\theta + cos^4\theta)$$

$$\overline{S}_{11} = S_{11}cos^4\theta + (2S_{12} + S_{66})sin^2\theta cos^2\theta + S_{22}sin^4\theta$$

$$\overline{S}_{22} = S_{11}sin^4\theta + (2S_{12} + S_{66})sin^2\theta cos^2\theta + S_{22}cos^4\theta$$

$$\overline{S}_{16} = (2S_{11} - 2S_{12} - S_{66})\sin\theta\, cos^3\theta - (2S_{22} - 2S_{12} - S_{66})sin^3\theta \cos\theta$$

$$\overline{S}_{26} = (2S_{11} - 2S_{12} - S_{66})sin^3\theta \cos\theta - (2S_{22} - 2S_{12} - S_{66})\sin\theta\, cos^3\theta$$

$$\overline{S}_{66} = 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})sin^2\theta cos^2\theta + S_{66}(sin^4\theta + cos^4\theta)$$

3.6. Stress-Strain Relations for anisotropic Laminae

As a matter of fact, there is no difference between Solution for generally orthotropic laminae and those for anisotropic laminae.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

or in inverted form as

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{16} \\ S_{12} & S_{22} & S_{26} \\ S_{16} & S_{26} & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$

We can write:

$E_1 \; \varepsilon_1$

$$\varepsilon_2 = S_{12}\sigma_1 + S_{22}\sigma_2 + S_{26}\tau_{12}$$

Before, we defined that

$$\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1}$$ for $\sigma_1 = \sigma$ and all other stresses are zero

$$S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2}$$

But what can we say about S_{26} and S_{16}

$\eta_{i,ij}$ = Coefficient of mutual influence of the first kind that characterizes stretching in the i-direction caused by shear stress in the ij-plane

$$\eta_{i,ij} = \frac{\varepsilon_i}{\gamma_{ij}}$$ for $\tau_{ij} = \tau$ and all other stresses are zero

(first Lekhnitskii coefficient of mutual influence)

(shear-extension coupling coefficient)

$\eta_{ij,i}$ = Coefficient of mutual influence of the second kind characterizing shearing in the ij-plane caused by normal stress in i-direction.

$$\eta_{ij,i} = \frac{\gamma_{ij}}{\varepsilon_i}$$ for $\sigma_i = \sigma$ and all other stresses are zero

(second Lekhnitskij coefficient of mutual influence)

$\mu_{ij,kl}$ = Chentsov coefficient that characterizes the shearing strain in the kl-plane due to shearing stress in the ij-plane.

$$\mu_{ij,kl} = \frac{\gamma_{kl}}{\gamma_{ij}}$$ for $\tau_{ij} = \tau$ and all other stresses are zero

S_{ij} is symmetric $\Rightarrow \begin{cases} \dfrac{\mu_{ij,kl}}{G_{kl}} = \dfrac{\mu_{kl,ij}}{G_{ij}} \\[2ex] \dfrac{\eta_{ij,i}}{E_i} = \dfrac{\eta_{i,ij}}{G_{ij}} \end{cases}$

$$\gamma_{13} = \frac{\eta_{13,1}}{E_1}\sigma_1 + \frac{\eta_{13,2}}{E_2}\sigma_2 + \frac{\mu_{13,12}}{G_{12}}\tau_{12} = \frac{\eta_{1,13}\sigma_1 + \eta_{2,13}\sigma_2 + \mu_{12,13}\tau_{12}}{G_{13}}$$

$$\gamma_{23} = \frac{\eta_{1,23}\sigma_1 + \eta_{2,23}\sigma_2 + \mu_{12,23}\tau_{12}}{G_{23}}$$