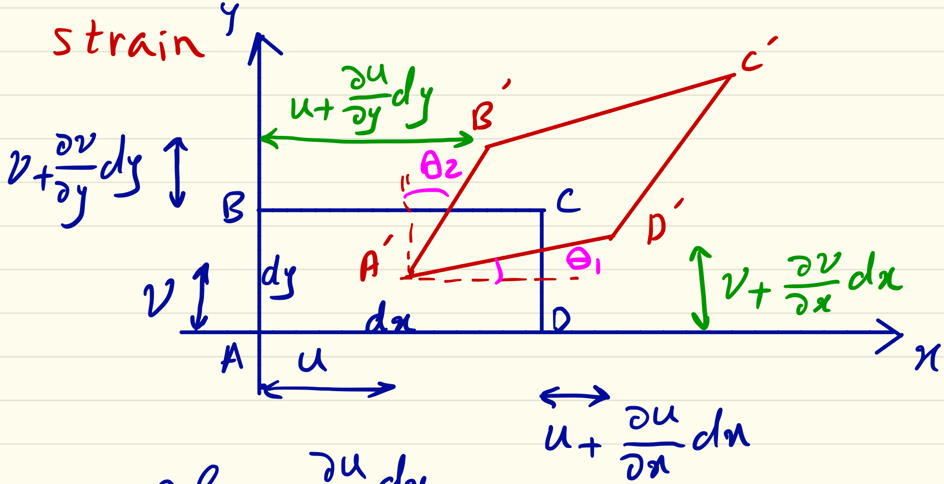
Composites Lesson 6 بسم الله الرحمن الرحيم

strain



Taylor expansion

$$\phi(x_0+\Delta x)=\phi(x_0)+\frac{d\phi}{dx}\Delta x+\dots$$

$$\varepsilon_x=\frac{\Delta l}{l}=\frac{\frac{\partial u}{\partial x}dx}{dx}=\frac{\partial u}{\partial x}$$

$$\varepsilon_y=\frac{\Delta l}{l}=\frac{\frac{\partial v}{\partial y}dy}{dy}=\frac{\partial v}{\partial y}$$

Shearin strain

$$\gamma_{xy}=\frac{\partial v/\partial x\, dx}{dx}+\frac{\partial u/\partial y\, dy}{dy}=\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\rightarrow \varepsilon_{xy}=\frac{1}{2}\gamma_{xy}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$$

3.2. Stress-Strain Relations

The generalized Hooke's law relating stress to strain can be written as:

$$G = \frac{E}{2(1+\nu)}$$

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{xz} = \frac{\tau_{xz}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$
\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{Bmatrix}
$$

Campliance matrix

$\{\varepsilon\} = [S]\{\sigma\}$ $\qquad$ $\varepsilon_i = S_{ij}\sigma_j$

$\{\sigma\} = [C]\{\varepsilon\}$

stiffness matrix

$[S] = [C]^{-1}$ $\qquad$ inverse of eachother

It can be improved that they are symmetric matrixs:

$$C_{ij} = C_{ji} \quad \& \quad S_{ij} = S_{ji}$$

and their components are referred to as elastic constants. Stiffness matrix is symmetric, and hence in general form it has 21 independent constants

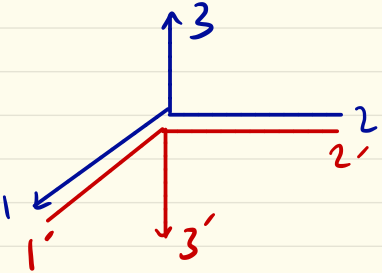
$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix}$$

anisotropic material

(without isotropy)

(21)

If there is one plane of material property symmetry (here, for example, the plane of symmetry is $z=0$), the stress-strain relations reduce.



$[C] = [C]'$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ S_{12} & S_{22} & S_{23} & 0 & 0 & S_{26} \\ S_{13} & S_{23} & S_{33} & 0 & 0 & S_{36} \\ 0 & 0 & 0 & S_{44} & S_{45} & 0 \\ 0 & 0 & 0 & S_{45} & S_{55} & 0 \\ S_{16} & S_{26} & S_{36} & 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix}$$

monoclinic material

(13 independent elastic constants)

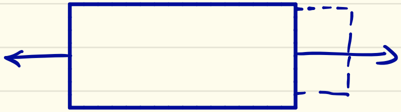
If there are two orthogonal planes of material property symmetry for a material, symmetry will exist relative to a third mutually orthogonal plane.

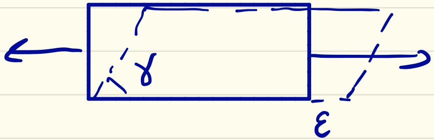
$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix}$$

orthotropic materials

(9 constats)

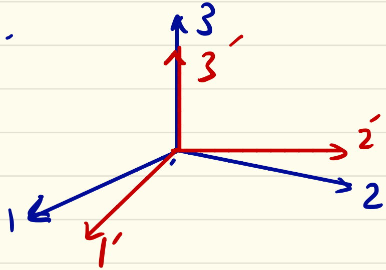
Note that there is no intraction between normal stresses $\sigma_1, \sigma_2, \sigma_3$ and shearing strains $\gamma_{23}, \gamma_{31}, \gamma_{12}$




γ
ε

If at every point of a material there is one plane in which the mecanical properties are equal in all directions, then the material is called transversely isotropic.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & (C_{11} - C_{12})/2 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix}$$



1-2: indefferent plane

Transversely Isotropic material

(5 constants)

If there is an infinite number of planes of indefferent direction in them:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & (C_{11}-C_{12})/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & (C_{11}-C_{12})/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & (C_{11}-C_{12})/2 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix}$$

Isotropic materials

(2 constants)

2 indefferent plane $\rightarrow$ infinit number

one of the major objectives in studying the strain-stress relations is to obtain the deformation response of the loaded body.

$$\varepsilon_1 = S_{11}\sigma_1 + S_{12}\sigma_2 + S_{13}\sigma_3 + S_{14}\tau_{23} + S_{15}\tau_{31} + S_{16}\tau_{12}$$

$$\vdots$$

$$\gamma_{12} = S_{16}\sigma_1 + S_{26}\sigma_2 + S_{36}\sigma_3 + S_{46}\tau_{23} + S_{56}\tau_{31} + S_{66}\tau_{12}$$

Accordingly, for an applied uniaxial stress $\sigma_1 = \sigma$ (all other stresses are zero)

$$\varepsilon_1 = S_{11}\sigma \;,\quad \varepsilon_2 = S_{12}\sigma \;,\quad \varepsilon_3 = S_{13}\sigma$$

$$\gamma_{23} = S_{14}\sigma \;,\quad \gamma_{31} = S_{15}\sigma \;,\quad \gamma_{12} = S_{16}\sigma$$

It means that with a stress we have many deformation. on the other hand, because $S_{11} \neq S_{22} \neq S_{33}$

the deformation of the body in each direction is different.

EXTENSION-EXTENSION COUPLING

EXTENSION

SHEAR-EXTENSION COUPLING

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{13} & S_{23} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{14} & S_{24} & S_{34} & S_{44} & S_{45} & S_{46} \\ S_{15} & S_{25} & S_{35} & S_{45} & S_{55} & S_{56} \\ S_{16} & S_{26} & S_{36} & S_{46} & S_{56} & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix}$$

SHEAR

SHEAR-SHEAR COUPLING

S_{11}, S_{22}, S_{33}: respose in the same direction

S_{12}, S_{13}, S_{23}: coupling between dissimilar normal stresses and normal strains. (Poisson effect)

The only Coupling that exists for an isotropic material is extension_extension Coupling.

3.3_ Stiffness, Compliance, and Engineering Constants for orthotropic materials.

Engineering constants are measured in simple tests (Young and shearing moduli, and Poisson's ratios, and some others

$$[S_{ij}] = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}$$

(10)

$$S_{ij} = S_{ji} \Rightarrow \frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j}$$

$\{\varepsilon\} = [S]\{\sigma\}$, $\{\sigma\} = [C]\{\varepsilon\}$ $\hookrightarrow$?

$$C_{11} = \frac{S_{22}S_{33} - S_{23}{}^2}{S}$$

$$C_{12} = \frac{S_{13}S_{23} - S_{12}S_{33}}{S}$$

$$C_{22} = \frac{S_{33}S_{11} - S_{13}{}^2}{S}$$

$$C_{13} = \frac{S_{12}S_{23} - S_{13}S_{22}}{S}$$

$$C_{33} = \frac{S_{11}S_{22} - S_{12}{}^2}{S}$$

$$C_{23} = \frac{S_{12}S_{13} - S_{23}S_{11}}{S}$$

$$C_{44} = \frac{1}{S_{44}}, \quad C_{55} = \frac{1}{S_{55}}, \quad C_{66} = \frac{1}{S_{66}}$$

$$S = S_{11}S_{22}S_{33} - S_{11}S_{23}^2 - S_{22}S_{13}^2 - S_{33}S_{12}^2 + 2S_{12}S_{23}S_{13}$$

$$C_{11} = \frac{1 - \nu_{23}\nu_{32}}{E_2 E_3 \Delta}, \qquad C_{22} = \frac{1 - \nu_{13}\nu_{31}}{E_1 E_3 \Delta}, \qquad C_{33} = \frac{1 - \nu_{12}\nu_{21}}{E_1 E_2 \Delta}$$

$$C_{12} = \frac{\nu_{21} + \nu_{31}\nu_{23}}{E_2 E_3 \Delta} = \frac{\nu_{12} + \nu_{32}\nu_{13}}{E_1 E_3 \Delta}$$

$$C_{13} = \frac{\nu_{31} + \nu_{21}\nu_{32}}{E_2 E_3 \Delta} = \frac{\nu_{13} + \nu_{12}\nu_{23}}{E_1 E_2 \Delta}$$

$$C_{23} = \frac{\nu_{32} + \nu_{12}\nu_{31}}{E_1 E_3 \Delta} = \frac{\nu_{23} + \nu_{21}\nu_{13}}{E_1 E_2 \Delta}$$

$$C_{44} = G_{23}, \qquad C_{55} = G_{31}, \qquad C_{66} = G_{16}$$

$$\Delta = \frac{1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{31}\nu_{13} - 2(\nu_{21}\nu_{32}\nu_{13})}{E_1 E_2 E_3}$$