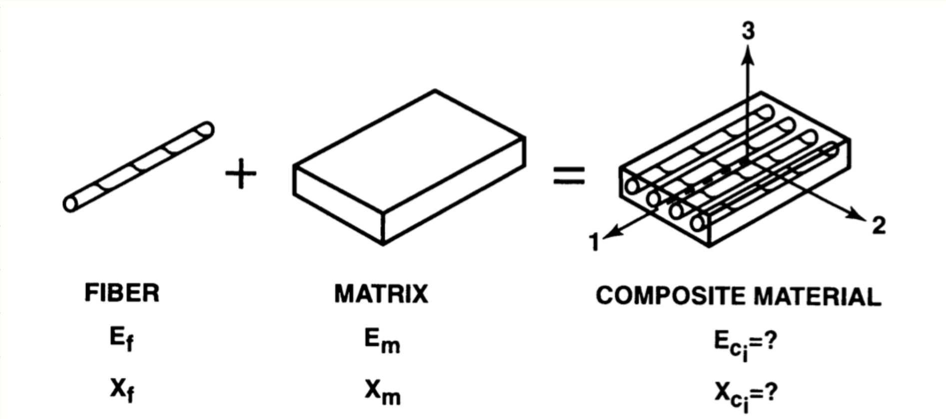
Composites Lesson 4 بسم الله الرحمن الرحيم

Chapter 2- Micromecanical Behavior of a Lamina

2.1- Introduction

The basic question is: how can the Stiffnesses and strength of a composite material be varied by changing the proportion of fibers to matrix?



$$C_{ij} = C_{ij}(E_f, \nu_f, v_f, E_m, \nu_m, v_m)$$

E_f = Young's modulus for fiber

ν_f = Poisson's ration " "

$$v_f = \frac{\text{Volume of fiber}}{\text{Total Volume of Composite material}}$$

An additional objective of micromecanics approach is to determine the strengths of the composite:

$$X_i = X_i(X_{if}, v_f, X_{im}, v_m)$$

$X_i = X, Y, S =$ Composite Material Strengths

$X_{if} = X_f, Y_f, S_f =$ Fiber Strengths

Some basic approaches to the micromecanics of composites:

1- Mechanics of Materials

2- experimental approach

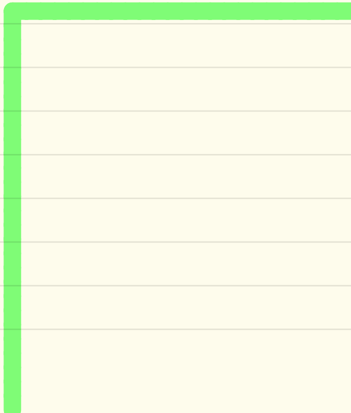
3- Elasticity (exact solution)

4- Semi-experimental approach

Basic restrictions on the composite material that can be treated here are:

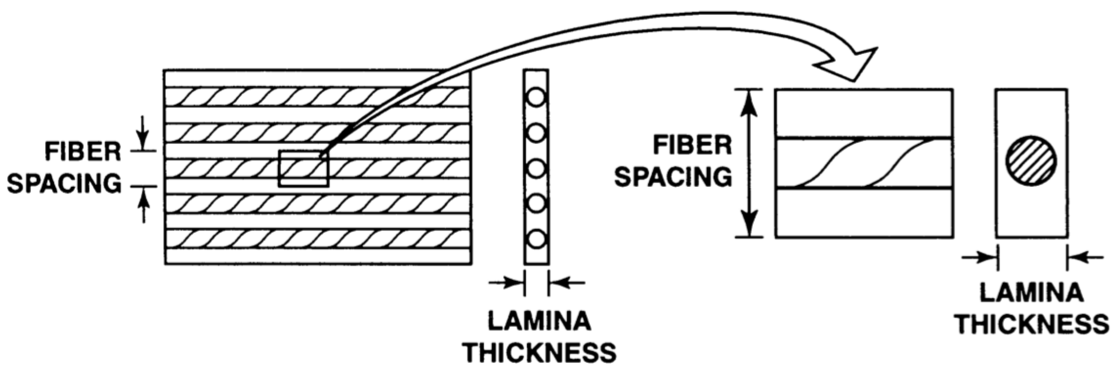
- The Lamina is
  - initially stress-free
  - linearly elastic
  - macroscopically homogeneous
  - macroscopically orthotropic
- The fibers are
  - homogeneous
  - linearly elastic
  - isotropic
  - regularly spaced
  - perfectly aligned
  - perfectly bonded

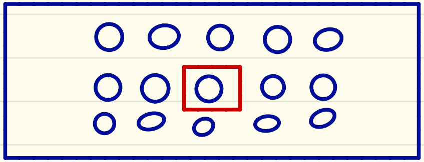
- The matrix is
  - homogeneause
  - isotropic
  - linearly elastic
  - void-free

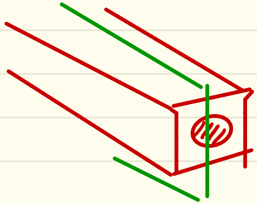


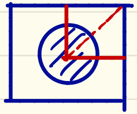
RVE (Representative Volume Element)

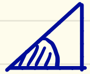
RVE is the smallest region or piece of material over which the stresses and strains can be regarded as macroscopically uniform and yet the volume still has the correct proportions of fiber and matrix.



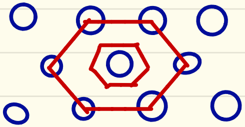


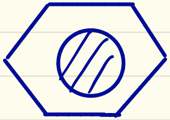




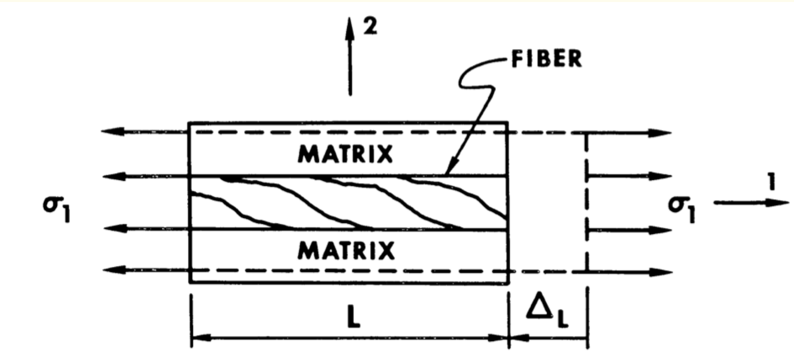


RVE



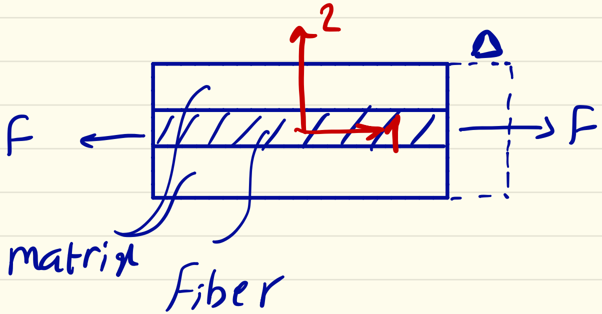


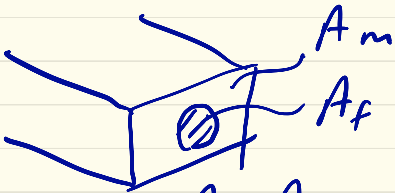
30°



2.2- mechanics of materials approach to stiffness

2.2-1- Determination of E_1





$$A = A_m + A_f$$

$$V = V_m + V_f$$

$$1 = v_m + v_f$$

$$\Delta_c = \Delta_f = \Delta_m \longrightarrow \varepsilon_c = \varepsilon_m = \varepsilon_f$$

Load sharing is as simple spring-in-parallel

$$F_c = F_m + F_f \longrightarrow \sigma_c \cdot A = \sigma_m A_m + \sigma_f A_f$$

Defination

$$v_f = \frac{V_f}{V_c}$$ Fiber-volume fraction

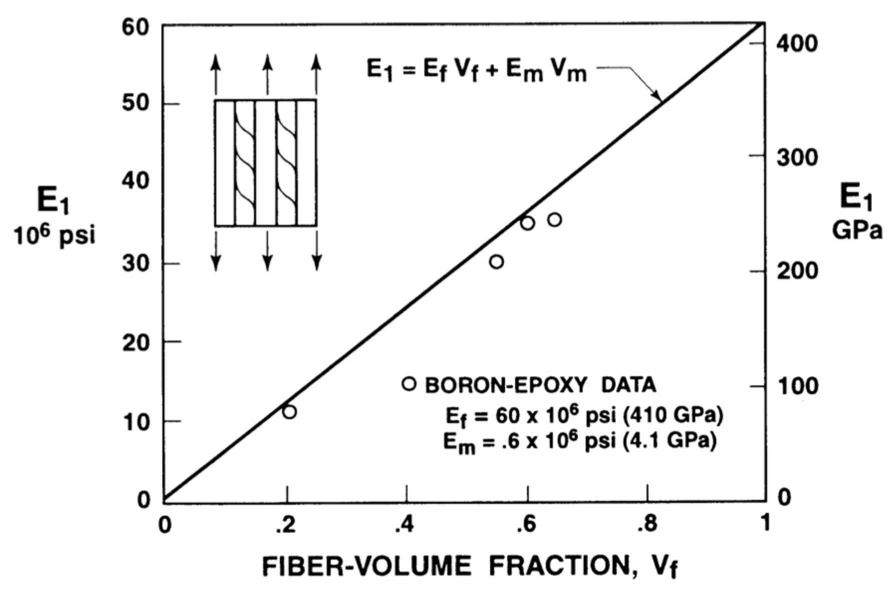
$$v_m = 1 - v_f$$

$$\sigma_c = \sigma_f \nu_f + \sigma_m (1 - \nu_f)$$

$$E_{c1} = E_1 = \frac{\sigma_c}{\varepsilon_c} = \frac{\sigma_f \nu_f + \sigma_m (1-\nu_f)}{\varepsilon_c} = \frac{\sigma_f}{\varepsilon_c} \nu_f + \frac{\sigma_m}{\varepsilon_c} (1-\nu_f)$$

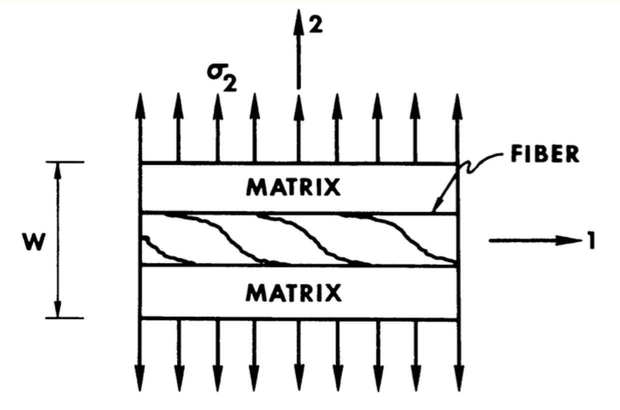
$$\boxed{E_1 = E_f \nu_f + E_m (1-\nu_f)}$$

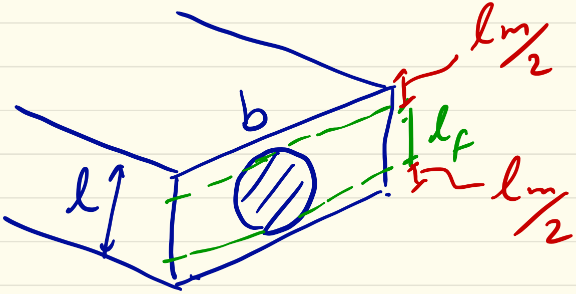
which is known as the rule of mixture.



E1 = Ef Vf + Em Vm
E1
10^6 psi
E1
GPa
60
50
40
30
20
10
0
400
300
200
100
0
0
.2
.4
.6
.8
1
BORON-EPOXY DATA
Ef = 60 x 10^6 psi (410 GPa)
Em = .6 x 10^6 psi (4.1 GPa)
FIBER-VOLUME FRACTION, Vf

2.2.2 Determination of E_2





springs-in-series

$$F_c = F_m = F_f \longrightarrow \sigma_c = \sigma_m = \sigma_f$$

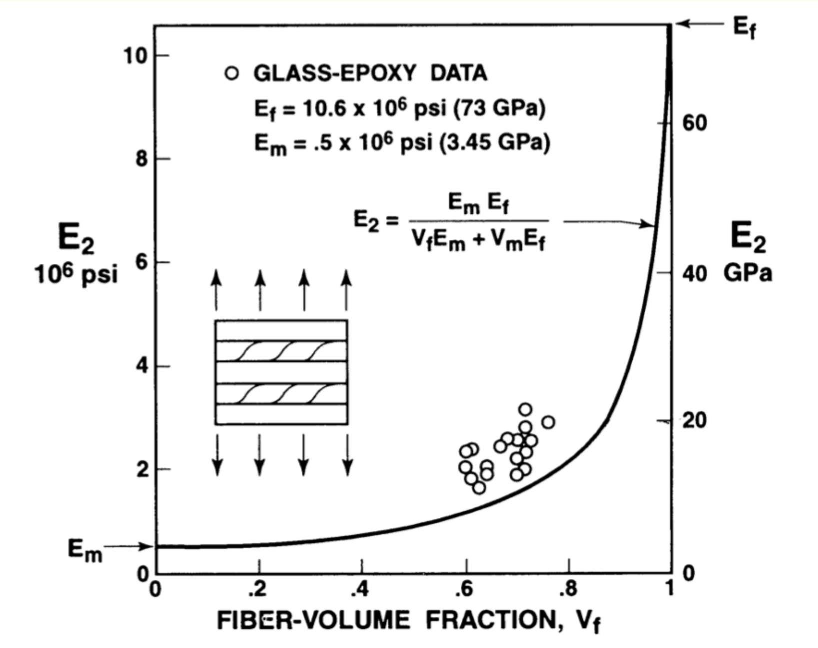
$$\Delta_c = \Delta_m + \Delta_f \rightarrow \varepsilon_c \cdot l = \varepsilon_f \cdot l_f + \varepsilon_m l_m$$

$$(*) \quad \varepsilon_c = \varepsilon_f v_f + \varepsilon_m (1 - v_f)$$

$$\frac{\cancel{\sigma_{2c}}}{E_{2c}} = \frac{\cancel{\sigma_{2f}}}{E_f} v_f + \frac{\cancel{\sigma_{2m}}}{E_m} (1 - v_f)$$

$$\boxed{\frac{1}{E_{2c}} = \frac{1}{E_{2f}} v_f + \frac{1}{E_{2m}} (1 - v_f)}$$

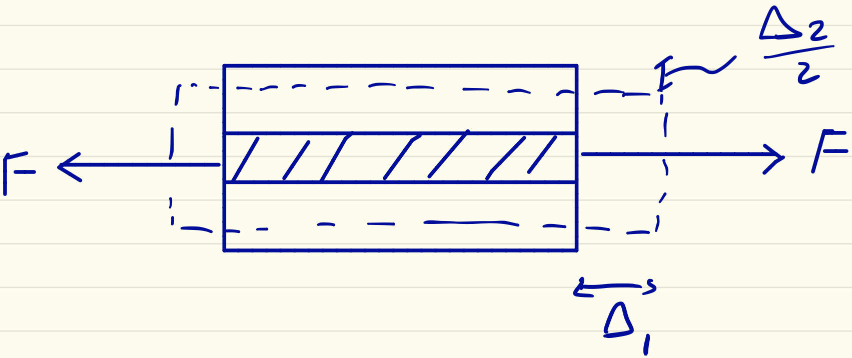
It's an understimate approach and it's not precise.


GLASS-EPOXY DATA
$E_f = 10.6 \times 10^6$ psi (73 GPa)
$E_m = .5 \times 10^6$ psi (3.45 GPa)
$E_2 = \frac{E_m E_f}{V_f E_m + V_m E_f}$
E_2
10^6 psi
E_2
GPa
E_f
E_m
0
.2
.4
.6
.8
1
0
2
4
6
8
10
0
20
40
60
FIBER-VOLUME FRACTION, V_f

2.2.3- Determination of ν_{12}

So-called major Poisson's ration is defined as;

$$\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1}$$



$(*) \rightarrow \quad \varepsilon_{2c} = \varepsilon_{2f} V_f + \varepsilon_{2m}(1-V_f)$

$$\cancel{\varepsilon_{1c}}\, \nu_c = \cancel{\varepsilon_{1f}}\, \nu_f V_f + \cancel{\varepsilon_{1m}}\, \nu_m (1-V_f)$$

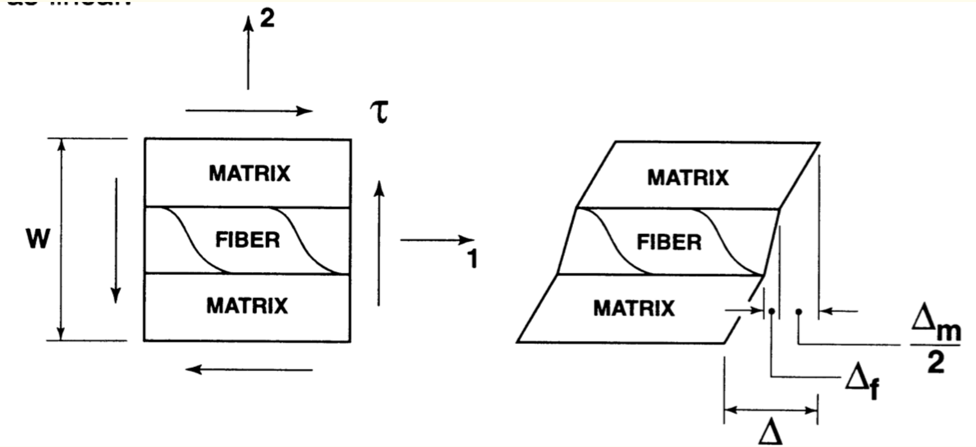
$$\boxed{\nu_c = \nu_f v_f + \nu_m (1 - v_f)}$$

rule of mixture
(Good prediction)

2.2.4 Determination of G_{12}

in-plane shear modulus of a lamina

$$\gamma = \frac{\tau}{G}$$



After abit of math, the following equation results

$$\boxed{\frac{1}{G_{12}} = \frac{1}{G_f} V_f + \frac{1}{G_m}(1-V_f)}$$

2.3 The Halpin-Tsai Equations

Halpin and Tsai developed an interpolation procedure that the result is simple used and quite accurate:

$$E_{1C} = E_f v_f + E_m v_m$$

$$\nu_{12} = \nu_f v_f + \nu_m v_m$$

$$M = M_m \frac{1 + \xi \eta v_f}{1 - \eta v_f}$$

$$\eta = \frac{M_f/M_m - 1}{M_f/M_m + \xi}$$

M = Composite material modulus E_2, G_{12} or ν_{23}

M_f = Corresponding fiber moduluse E_f, G_f or ν_f

M_m = " Matrix " E_m, G_m or ν_m

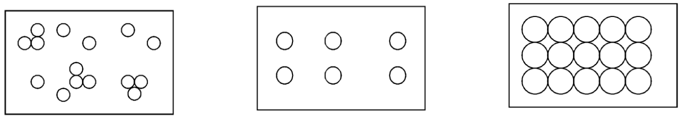
ξ is a measure of fiber reinforcement to the composite material that depends on the fiber geometry, packing geometry, and loading condition.

2.4- Elastisity Solution

$$E_2 = 2\left[1 - \nu_f + (\nu_f - \nu_m)V_m\right]\left[(1 - C)\frac{(K_f(2K_m + G_m) - G_m(K_f - K_m)V_m)}{(2K_m + G_m) + 2(K_f - K_m)V_m} + C\frac{(K_f(2K_m + G_f) + G_f(K_m - K_f)V_m)}{(2K_m + G_f) - 2(K_m - K_f)V_m}\right]$$

$$\upsilon_{12} = \left[(1 - C)\frac{K_f\nu_f(2K_m + G_m)V_f + K_m\nu_m(2K_f + G_m)V_m}{K_f(2K_m + G_m) - G_m(K_f - K_m)V_m} + C\frac{K_m\nu_m(2K_f + G_f)V_m + K_f\nu_f(2K_m + G_f)V_f}{K_f(2K_m + G_m) + G_f(K_m - K_f)V_m}\right]$$

$$G_{12} = \left[(1 - C)G_m\frac{2G_f - (G_f - G_m)V_m}{2G_m + (G_f - G_m)V_m} + CG_f\frac{(G_f + G_m) - (G_f - G_m)V_m}{(G_f + G_m) + (G_f - G_m)V_m}\right]$$



واقعی　　　C=0　　　C=1

همچنین :

$$K_m = \frac{E_m}{2(1-\nu_m)} \quad K_f = \frac{E_f}{2(1-\nu_f)}$$

$$G_m = \frac{E_m}{2(1+\nu_m)} \quad G_f = \frac{E_f}{2(1+\nu_f)}$$