Chapter VII Third-order Theory of Laminated Composite Plates

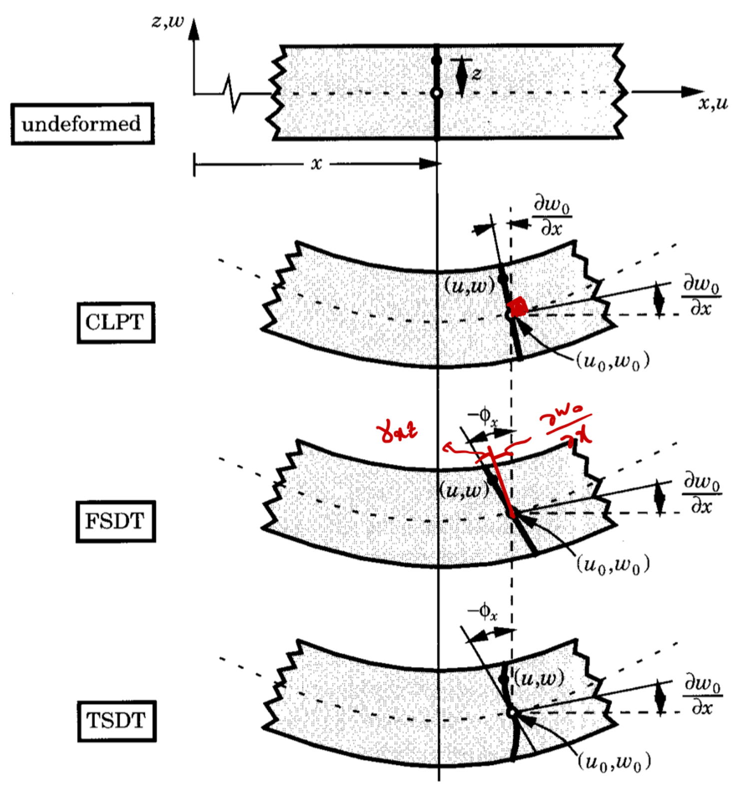
7.1- Introduction

تئوری های درجه بالا سیستماتیک مسأله را بهتر مدل می‌کنند و احتیاج به ضریب اصلاح برشی (K) ندارند همچنین تنش بین لایه ای را بهتر بیان می‌کنند اما هزینه محاسباتی بیشتری دارند لذا باید دقت شود که تنها وقتی نیاز است از این تئوری ها استفاده شود.

علتی که توزیع جابجائی درجه ۳ درنظر گرفته شده این است که میتوان به توزیع تنش برشی درجه ۲ در ضخامت رسید که به واقعیت نزدیک است. لذا درنظر گرفتن درجات بالاتر برای توزیع جابجائی در ضخامت بنظر مورد نیاز نیست.

7.2 A Third-order Plate Theory

7.2.1 Displacement Field



$$
\begin{aligned}
u &= u_0 + z\phi_x + z^2\theta_x + z^3\lambda_x \\
v &= v_0 + z\phi_x + z^2\theta_y + z^3\lambda_y \\
w &= w_0
\end{aligned}
\qquad (7.2\text{-}1)
$$

$$
\begin{aligned}
&u_0 = u(x,y,0,t), \qquad v_0 = v(x,y,0,t), \qquad w_0 = w(x,y,0,t) \\
&\phi_x = \left(\frac{\partial u}{\partial z}\right)_{z=0}, \qquad \phi_y = \left(\frac{\partial v}{\partial z}\right)_{z=0} \qquad 2\theta_x = \left(\frac{\partial^2 u}{\partial z^2}\right)_{z=0}, \\
&2\theta_y = \left(\frac{\partial^2 v}{\partial z^2}\right)_{z=0} \qquad 6\lambda_x = \left(\frac{\partial^3 u}{\partial z^3}\right)_{z=0}, \qquad 6\lambda_y = \left(\frac{\partial^3 v}{\partial z^3}\right)_{z=0}
\end{aligned}
\qquad (7.2\text{-}2)
$$

اگر در نظر بگیریم که در سطوح بالا و پایین تنش برشی خارجی نداشته باشیم، می توان گفت:

$$
\sigma_{xz}(x,y,\pm h/2,t) = 0, \quad \sigma_{yz}(x,y,\pm h/2,t) = 0 \qquad (7.2\text{-}3)
$$

$\Rightarrow$

$$
\begin{aligned}
0 = \sigma_{xz}(x,y,\pm h/2,t) &= Q_{55}\gamma_{xz}(x,y,\pm h/2,t) + Q_{45}\gamma_{yz}(x,y,\pm h/2,t), \\
0 = \sigma_{yz}(x,y,\pm h/2,t) &= Q_{45}\gamma_{xz}(x,y,\pm h/2,t) + Q_{44}\gamma_{yz}(x,y,\pm h/2,t)
\end{aligned}
\qquad (7.2\text{-}4)
$$

$$\phi_x + \frac{\partial w_0}{\partial x} + \left(-h\theta_x + \frac{3h^2}{4}\lambda_x\right) = 0, \quad \phi_x + \frac{\partial w_0}{\partial x} + \left(h\theta_x + \frac{3h^2}{4}\lambda_x\right) = 0$$

$$\phi_y + \frac{\partial w_0}{\partial y} + \left(-h\theta_y + \frac{3h^2}{4}\lambda_y\right) = 0, \quad \phi_y + \frac{\partial w_0}{\partial y} + \left(h\theta_y + \frac{3h^2}{4}\lambda_y\right) = 0$$

or

$$\lambda_x = -\frac{4}{3h^2}\left(\phi_x + \frac{\partial w_0}{\partial x}\right), \quad \theta_x = 0; \quad \lambda_y = -\frac{4}{3h^2}\left(\phi_y + \frac{\partial w_0}{\partial y}\right), \quad \theta_y = 0 \qquad (7.2\text{-}5)$$

$\Rightarrow$

$$\boxed{\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\phi_x(x,y,t) - \underbrace{\frac{4}{3h^2}}_{c_1} z^3\left(\phi_x + \frac{\partial w_0}{\partial x}\right) \\ v(x,y,z,t) &= v_0(x,y,t) + z\phi_y(x,y,t) - \frac{4}{3h^2} z^3\left(\phi_y + \frac{\partial w_0}{\partial y}\right) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned}} \qquad (7.2\text{-}6)$$

TSDT

(در دو سطح بدون تنش برشی)

7.2-2 Strains

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} + z^3 \begin{Bmatrix} \varepsilon_{xx}^{(3)} \\ \varepsilon_{yy}^{(3)} \\ \gamma_{xy}^{(3)} \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \end{Bmatrix} + z^2 \begin{Bmatrix} \gamma_{yz}^{(2)} \\ \gamma_{xz}^{(2)} \end{Bmatrix} \tag{7.2-7}$$

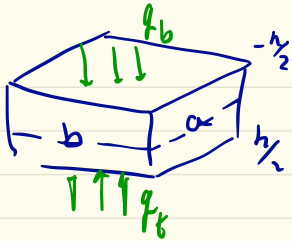
با فرض $c_1 = \frac{4}{3h^2}$ و $c_2 = 3c_1$ داریم:

$$\begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \end{Bmatrix} = \begin{Bmatrix} \phi_y + \frac{\partial w_0}{\partial y} \\ \phi_x + \frac{\partial w_0}{\partial x} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_{xx}^{(3)} \\ \varepsilon_{yy}^{(3)} \\ \gamma_{xy}^{(3)} \end{Bmatrix} = -c_1 \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2} \\ \frac{\partial \phi_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2} \\ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} + 2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^{(2)} \\ \gamma_{xz}^{(2)} \end{Bmatrix} = -c_2 \begin{Bmatrix} \phi_y + \frac{\partial w_0}{\partial y} \\ \phi_x + \frac{\partial w_0}{\partial x} \end{Bmatrix} \tag{7.2-8}$$

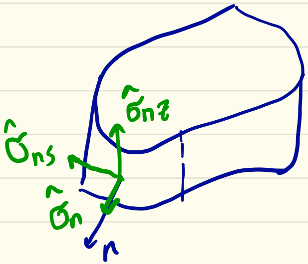
7.2-3 Equation of Motion



$$\delta U = \int_{\Omega_0} \left\{ \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\sigma_{xx} \left(\delta\varepsilon_{xx}^{(0)} + z\delta\varepsilon_{xx}^{(1)} - c_1 z^3 \delta\varepsilon_{xx}^{(3)} \right) + \sigma_{yy} \left(\delta\varepsilon_{yy}^{(0)} + z\delta\varepsilon_{yy}^{(1)} - c_1 z^3 \delta\varepsilon_{yy}^{(3)} \right) + \sigma_{xy} \left(\delta\gamma_{xy}^{(0)} + z\delta\gamma_{xy}^{(1)} - c_1 z^3 \delta\gamma_{xy}^{(3)} \right) + \sigma_{xz} \left(\delta\gamma_{xz}^{(0)} + z^2\delta\gamma_{xz}^{(2)} \right) + \sigma_{yz} \left(\delta\gamma_{yz}^{(0)} + z^2\delta\gamma_{yz}^{(2)} \right) \right] dz \right\} dxdy$$

(7.2-9)

$$= \int_{\Omega_0} \Big(N_{xx}\delta\varepsilon_{xx}^{(0)} + M_{xx}\delta\varepsilon_{xx}^{(1)} - c_1 P_{xx}\delta\varepsilon_{xx}^{(3)} + N_{yy}\delta\varepsilon_{yy}^{(0)} + M_{yy}\delta\varepsilon_{yy}^{(1)} - c_1 P_{yy}\delta\varepsilon_{yy}^{(3)} + N_{xy}\delta\gamma_{xy}^{(0)} + M_{xy}\delta\gamma_{xy}^{(1)} - c_1 P_{xy}\delta\gamma_{xy}^{(3)} + Q_x\delta\gamma_{xz}^{(0)} - c_2 R_x\delta\gamma_{xz}^{(2)} + Q_y\delta\gamma_{yz}^{(0)} - c_2 R_y\delta\gamma_{yz}^{(2)} \Big) dxdy$$



$$\delta V = -\int_{\Omega_0} \left[q_b(x,y)\delta w(x,y,-\frac{h}{2}) + q_t(x,y)\delta w(x,y,\frac{h}{2}) \right] dxdy - \int_\Gamma \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\hat{\sigma}_{nn} \left(\delta u_n + z\delta\phi_n - c_1 z^3 \delta\varphi_n \right) + \hat{\sigma}_{ns} \left(\delta u_s + z\delta\phi_s - c_1 z^3 \delta\varphi_{ns} \right) + \hat{\sigma}_{nz}\delta w_0 \right] dz d\Gamma$$

(7.2-10)

$$= -\int_{\Omega_0} q\delta w_0 dxdy - \int_\Gamma \left(\hat{N}_{nn}\delta u_n + \hat{M}_{nn}\delta\phi_n - c_1 \hat{P}_{nn}\delta\varphi_n + \hat{N}_{ns}\delta u_s + \hat{M}_{ns}\delta\phi_s - c_1 \hat{P}_{ns}\delta\varphi_{ns} + \hat{Q}_n\delta w_0 \right) d\Gamma$$

$$
\begin{aligned}
\delta K = \int_{\Omega_0} \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho_0 \Big[& \left(\dot{u}_0 + z\dot{\phi}_x - c_1 z^3 \dot{\varphi}_x\right)\left(\delta\dot{u}_0 + z\delta\dot{\phi}_x - c_1 z^3 \delta\dot{\varphi}_x\right) \\
& + \left(\dot{v}_0 + z\dot{\phi}_y - c_1 z^3 \dot{\varphi}_y\right)\left(\delta\dot{v}_0 + z\delta\dot{\phi}_y - c_1 z^3 \delta\dot{\varphi}_y\right) + \dot{w}_0 \delta\dot{w}_0 \Big] dv \\
= \int_{\Omega_0} \Big[& \left(I_0 \dot{u}_0 + I_1 \dot{\phi}_x - c_1 I_3 \dot{\varphi}_x\right)\delta\dot{u}_0 + \left(I_1 \dot{u}_0 + I_2 \dot{\phi}_x - c_1 I_4 \dot{\varphi}_x\right)\delta\dot{\phi}_x \\
& - c_1 \left(I_3 \dot{u}_0 + I_4 \dot{\phi}_x - c_1 I_6 \dot{\varphi}_x\right)\delta\dot{\varphi}_x + \left(I_0 \dot{v}_0 + I_1 \dot{\phi}_y - c_1 I_3 \dot{\varphi}_y\right)\delta\dot{v}_0 \\
& + \left(I_1 \dot{v}_0 + I_2 \dot{\phi}_y - c_1 I_4 \dot{\varphi}_y\right)\delta\dot{\phi}_y - c_1 \left(I_3 \dot{u}_0 + I_4 \dot{\phi}_y - c_1 I_6 \dot{\varphi}_y\right)\delta\dot{\varphi}_y \Big] dxdy
\end{aligned}
$$

(7.2-11)

Ω_0 لایه میانی است . همینی

$$
\begin{Bmatrix} N_{\alpha\beta} \\ M_{\alpha\beta} \\ P_{\alpha\beta} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\alpha\beta} \begin{Bmatrix} 1 \\ z \\ z^3 \end{Bmatrix} dz, \quad \begin{Bmatrix} Q_\alpha \\ R_\alpha \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\alpha z} \begin{Bmatrix} 1 \\ z^2 \end{Bmatrix} dz
$$

(7.2-12)

$$
I_i = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho_0 (z)^i \, dz \quad (i = 0, 1, 2, \cdots, 6)
$$

با استفاده از روابط فوق معادلات حاکم چنین می‌شوند.

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot{u}_0 + J_1 \ddot{\phi}_x - c_1 I_3 \frac{\partial \ddot{w}_0}{\partial x} \quad a$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = I_0 \ddot{v}_0 + J_1 \ddot{\phi}_y - c_1 I_3 \frac{\partial \ddot{w}_0}{\partial y} \quad b$$

$$\begin{aligned}
&\frac{\partial \bar{Q}_x}{\partial x} + \frac{\partial \bar{Q}_y}{\partial y} + \frac{\partial}{\partial x}\left(N_{xx}\frac{\partial w_0}{\partial x} + N_{xy}\frac{\partial w_0}{\partial y}\right) + \frac{\partial}{\partial y}\left(N_{xy}\frac{\partial w_0}{\partial x} + N_{yy}\frac{\partial w_0}{\partial y}\right) \\
&+ c_1\left(\frac{\partial^2 P_{xx}}{\partial x^2} + 2\frac{\partial^2 P_{xy}}{\partial x \partial y} + \frac{\partial^2 P_{yy}}{\partial y^2}\right) + q = I_0 \ddot{w}_0 - c_1^2 I_6\left(\frac{\partial^2 \ddot{w}_0}{\partial x^2} + \frac{\partial^2 \ddot{w}_0}{\partial y^2}\right) \quad c \\
&+ c_1\left[I_3\left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) + J_4\left(\frac{\partial \ddot{\phi}_x}{\partial x} + \frac{\partial \ddot{\phi}_y}{\partial y}\right)\right]
\end{aligned}$$

$$\frac{\partial \bar{M}_{xx}}{\partial x} + \frac{\partial \bar{M}_{xy}}{\partial y} - \bar{Q}_x = J_1 \ddot{u}_0 + K_2 \ddot{\phi}_x - c_1 J_4 \frac{\partial \ddot{w}_0}{\partial x} \quad d$$

$$\frac{\partial \bar{M}_{xy}}{\partial x} + \frac{\partial \bar{M}_{yy}}{\partial y} - \bar{Q}_y = J_1 \ddot{v}_0 + K_2 \ddot{\phi}_y - c_1 J_4 \frac{\partial \ddot{w}_0}{\partial y} \quad e$$

(7.2-13)

که در آنها

$$\bar{M}_{\alpha\beta} = M_{\alpha\beta} - c_1 P_{\alpha\beta}\ (\alpha, \beta = 1, 2, 6); \quad \bar{Q}_\alpha = Q_\alpha - c_2 R_\alpha\ (\alpha = 4, 5)$$

$$I_i = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \rho^{(k)} (z)^i \, dz \quad (i = 0, 1, 2, \cdots, 6)$$

$$J_i = I_i - c_1 I_{i+2}, \; K_2 = I_2 - 2c_1 I_4 + c_1^2 I_6, \; c_1 = \frac{4}{3h^2}, \; c_2 = 3c_1$$

(7.2-14)

$$\text{Primary Variables}: \quad u_n,\ u_s,\ w_0,\ \frac{\partial w_0}{\partial n},\ \phi_n,\ \phi_s$$

$$\text{Secondary Variables}: \quad N_{nn},\ N_{ns},\ \bar{V}_n,\ P_{nn},\ \bar{M}_{nn},\ \bar{M}_{ns}$$

(7.2-15)

که در آن

$$\begin{aligned}\bar{V}_n &\equiv c_1\left[\left(\frac{\partial P_{xx}}{\partial x}+\frac{\partial P_{xy}}{\partial y}\right)n_x+\left(\frac{\partial P_{xy}}{\partial x}+\frac{\partial P_{yy}}{\partial y}\right)n_y\right]\\ &\quad - c_1\left[\left(I_3\ddot{u}_0+J_4\ddot{\phi}_x-c_1 I_6\frac{\partial \ddot{w}_0}{\partial x}\right)n_x+\left(I_3\ddot{v}_0+J_4\ddot{\phi}_y-c_1 I_6\frac{\partial \ddot{w}_0}{\partial y}\right)n_y\right]\\ &\quad + (\bar{Q}_x n_x+\bar{Q}_y n_y)+\mathcal{P}(w_0)+c_1\frac{\partial P_{ns}}{\partial s}\end{aligned}$$

(7.2-16)

$$\mathcal{P}(w_0) = \left(N_{xx}\frac{\partial w_0}{\partial x}+N_{xy}\frac{\partial w_0}{\partial y}\right)n_x+\left(N_{xy}\frac{\partial w_0}{\partial x}+N_{yy}\frac{\partial w_0}{\partial y}\right)n_y$$

$\Rightarrow$

$$\boxed{\begin{aligned}\begin{Bmatrix}\{N\}\\\{M\}\\\{P\}\end{Bmatrix} &= \begin{bmatrix}[A] & [B] & [E]\\ [B] & [D] & [F]\\ [E] & [F] & [H]\end{bmatrix}\begin{Bmatrix}\{\varepsilon^{(0)}\}\\\{\varepsilon^{(1)}\}\\\{\varepsilon^{(3)}\}\end{Bmatrix}\\ \begin{Bmatrix}\{Q\}\\\{R\}\end{Bmatrix} &= \begin{bmatrix}[A] & [D]\\ [D] & [F]\end{bmatrix}\begin{Bmatrix}\{\gamma^{(0)}\}\\\{\gamma^{(2)}\}\end{Bmatrix}\end{aligned}}$$

معادله فضای حالت TSDT (7.2-17)

$$(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \bar{Q}_{ij}^{(k)} \; (1, z, z^2, z^3, z^4, z^6) \; dz$$

$i,j = 1,2,6$ (7.2-18)

ماتریس های 3×3

$$(A_{ij}, D_{ij}, F_{ij}) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \bar{Q}_{ij}^{(k)} \; (1, z^2, z^4) \; dz$$

$i,j = 4,5$ (7.2-19)

ماتریس های 2×2

ماتریس های A و B و D را که قبلاً می‌شناختیم. اما ماتریس های E و F و H چنین خواهند بود

$$E_{ij} = \frac{1}{4} \sum_{k=1}^{N} \bar{Q}_{ij}^{(k)} \left[(z_{k+1})^4 - (z_k)^4 \right]$$

$$F_{ij} = \frac{1}{5} \sum_{k=1}^{N} \bar{Q}_{ij}^{(k)} \left[(z_{k+1})^5 - (z_k)^5 \right]$$

$$H_{ij} = \frac{1}{7} \sum_{k=1}^{N} \bar{Q}_{ij}^{(k)} \left[(z_{k+1})^7 - (z_k)^7 \right]$$

(7.2-20)

در معادلات حاکم فوق (7.2-13) با قرار دادن $C_1=0$ به معادلات FSDT خواهیم رسید.

7.3 Higher-order Laminate Stiffness characteristics

گاهی، تئوری درجه ۳ را چنین ساده‌سازی می‌کنند که از P_{xx} و P_{yy} و P_{xy} صرف‌نظر می‌کنند

اما R_x و R_y را در نظر می‌گیرند.

ماتریس سفتی هر لایه مانند قبل خواهد بود.

$$Q_{11}=\frac{E_1}{1-\nu_{12}\nu_{21}}\,,\quad Q_{12}=\frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}\,,\quad Q_{22}=\frac{E_2}{1-\nu_{12}\nu_{21}}$$
$$Q_{66}=G_{12},\ Q_{44}=G_{23},\ Q_{55}=G_{13} \qquad (7.2\text{-}21)$$

I. Single-Layer Plates

Single Isotropic Layer

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & \nu A_{11} & 0 \\ \nu A_{11} & A_{11} & 0 \\ 0 & 0 & \frac{1-\nu}{2} A_{11} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix}$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & \nu D_{11} & 0 \\ \nu D_{11} & D_{11} & 0 \\ 0 & 0 & \frac{1-\nu}{2} D_{11} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} + \begin{bmatrix} F_{11} & \nu F_{11} & 0 \\ \nu F_{11} & F_{11} & 0 \\ 0 & 0 & \frac{1-\nu}{2} F_{11} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(3)} \\ \varepsilon_{yy}^{(3)} \\ \gamma_{xy}^{(3)} \end{Bmatrix} \tag{11.3.4b}$$

(7.2-22)

$$\begin{Bmatrix} P_{xx} \\ P_{yy} \\ P_{xy} \end{Bmatrix} = \begin{bmatrix} F_{11} & \nu F_{11} & 0 \\ \nu F_{11} & F_{11} & 0 \\ 0 & 0 & \frac{1-\nu}{2} F_{11} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} + \begin{bmatrix} H_{11} & \nu H_{11} & 0 \\ \nu H_{11} & H_{11} & 0 \\ 0 & 0 & \frac{1-\nu}{2} H_{11} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(3)} \\ \varepsilon_{yy}^{(3)} \\ \gamma_{xy}^{(3)} \end{Bmatrix}$$

$$\begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = \frac{1-\nu}{2} \begin{bmatrix} A_{11} & 0 \\ 0 & A_{11} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \end{Bmatrix} + \frac{1-\nu}{2} \begin{bmatrix} D_{11} & 0 \\ 0 & D_{11} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^{(2)} \\ \gamma_{xz}^{(2)} \end{Bmatrix}$$

$$\begin{Bmatrix} R_y \\ R_x \end{Bmatrix} = \frac{1-\nu}{2} \begin{bmatrix} D_{11} & 0 \\ 0 & D_{11} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \end{Bmatrix} + \frac{1-\nu}{2} \begin{bmatrix} F_{11} & 0 \\ 0 & F_{11} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^{(2)} \\ \gamma_{xz}^{(2)} \end{Bmatrix}$$

Single Specially Orthotropic Layer

رابطه N و M مانند قبل خواهد بود.

$$\begin{Bmatrix} P_{xx} \\ P_{yy} \\ P_{xy} \end{Bmatrix} = \frac{h^5}{80} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} + \frac{h^7}{448} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(3)} \\ \varepsilon_{yy}^{(3)} \\ \gamma_{xy}^{(3)} \end{Bmatrix} \quad (7.2\text{-}23)$$

(11.3

$$\begin{Bmatrix} R_y \\ R_x \end{Bmatrix} = \frac{h^3}{12} \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \end{Bmatrix} + \frac{h^5}{80} \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^{(2)} \\ \gamma_{xz}^{(2)} \end{Bmatrix} \quad (11.3$$

Single Generally Orthotropic Layer

$$\begin{Bmatrix} P_{xx} \\ P_{yy} \\ P_{xy} \end{Bmatrix} = \begin{bmatrix} F_{11} & F_{12} & F_{16} \\ F_{12} & F_{22} & F_{26} \\ F_{16} & F_{26} & F_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} + \begin{bmatrix} H_{11} & H_{12} & H_{16} \\ H_{12} & H_{22} & H_{26} \\ H_{16} & H_{26} & H_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(3)} \\ \varepsilon_{yy}^{(3)} \\ \gamma_{xy}^{(3)} \end{Bmatrix}$$

(7.2-24)

$$\begin{Bmatrix} R_y \\ R_x \end{Bmatrix} = \begin{bmatrix} D_{44} & D_{45} \\ D_{45} & D_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \end{Bmatrix} + \begin{bmatrix} F_{44} & F_{45} \\ F_{45} & F_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^{(2)} \\ \gamma_{xz}^{(2)} \end{Bmatrix}$$

higher-order thermal stress resultants for this case are given by

$$\begin{Bmatrix} P_{xx}^T \\ P_{yy}^T \\ P_{xy}^T \end{Bmatrix} = \sum_{k=1}^{L} \int_{z_k}^{z_{k+1}} \begin{bmatrix} \bar{Q}_{11}^k & \bar{Q}_{12}^k & \bar{Q}_{16}^k \\ \bar{Q}_{12}^k & \bar{Q}_{22}^k & \bar{Q}_{26}^k \\ \bar{Q}_{16}^k & \bar{Q}_{26}^k & \bar{Q}_{66}^k \end{bmatrix}^{(k)} \begin{Bmatrix} \alpha_{xx} \\ \alpha_{yy} \\ 2\alpha_{xy} \end{Bmatrix}^{(k)} \Delta T z^3 dz \quad (7.2\text{-}25)$$

II Symmetric Laminates

N و M مانند قبل (7.2-22) خواهد بود.

Symmetric Laminates with Multiple Isotropic Layers

$$\begin{Bmatrix} P_{xx} \\ P_{yy} \\ P_{xy} \end{Bmatrix} = \begin{bmatrix} F_{11} & F_{12} & 0 \\ F_{12} & F_{11} & 0 \\ 0 & 0 & F_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} + \begin{bmatrix} H_{11} & H_{12} & 0 \\ H_{12} & H_{11} & 0 \\ 0 & 0 & H_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(3)} \\ \varepsilon_{yy}^{(3)} \\ \gamma_{xy}^{(3)} \end{Bmatrix} \qquad (7.2\text{-}26)$$

$$\begin{Bmatrix} R_y \\ R_x \end{Bmatrix} = \begin{bmatrix} D_{44} & 0 \\ 0 & D_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \end{Bmatrix} + \begin{bmatrix} F_{44} & 0 \\ 0 & F_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^{(2)} \\ \gamma_{xz}^{(2)} \end{Bmatrix}.$$

III Antisymmetric Laminates

$F_{16} = F_{26} = H_{16} = H_{26} = 0$ & $B_{ij}, F_{ij} \neq 0$

$$\begin{Bmatrix} P_{xx} \\ P_{yy} \\ P_{xy} \end{Bmatrix} = \begin{bmatrix} E_{11} & E_{12} & E_{16} \\ E_{12} & E_{22} & E_{26} \\ E_{16} & E_{26} & E_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \varepsilon_{xy}^{(0)} \end{Bmatrix} + \begin{bmatrix} F_{11} & F_{12} & 0 \\ F_{12} & F_{22} & 0 \\ 0 & 0 & F_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \varepsilon_{xy}^{(1)} \end{Bmatrix}$$
$$+ \begin{bmatrix} H_{11} & H_{12} & 0 \\ H_{12} & H_{22} & 0 \\ 0 & 0 & H_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(3)} \\ \varepsilon_{yy}^{(3)} \\ \varepsilon_{xy}^{(3)} \end{Bmatrix} \qquad (7.2\text{-}27)$$

$$\begin{Bmatrix} R_y \\ R_x \end{Bmatrix} = \begin{bmatrix} D_{44} & 0 \\ 0 & D_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{yz}^{(0)} \\ \varepsilon_{xz}^{(0)} \end{Bmatrix} + \begin{bmatrix} F_{44} & 0 \\ 0 & F_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{yz}^{(2)} \\ \varepsilon_{xz}^{(2)} \end{Bmatrix}$$