بسم الله الرحمن الرحیم

مواد مرکب

جلسه ۲۴

با قرار دادن رابطه (I) در رابطه (6.3-9b) داریم:

$$\frac{dw_0}{dx} = \frac{F_0 b a^2}{16 E_x^b I_y}\left[1-4\left(\frac{x}{a}\right)^2\right] + \frac{F_0 b}{2kG_{xz}^b bh} \qquad (\mathrm{II})$$

قسمت اول رابطه شیب همان ϕ_x است.

$$\frac{dw_0^b}{dx} = \frac{F_0 b a^2}{16 E_x^b I_y}\left[1-4\left(\frac{x}{a}\right)^2\right] = -\phi_x \qquad (\mathrm{III})$$

حال با توجه به (6.3-15) می توان گفت

$$w_0 = w_0^b + w_0^s \longrightarrow \frac{dw_0}{dx} = \frac{dw_0^b}{dx} + \frac{dw_0^s}{dx} \qquad (\mathrm{IV})$$

اما قبلاً گفتیم

$$\gamma_{xz} = 2\varepsilon_{xz} = \frac{\partial w_0}{\partial x} + \phi_x$$

حال

$$\mathrm{IV} \Rightarrow \frac{dw_0^s}{dx} = \frac{dw_0}{dx} - \frac{dw_0^b}{dx} = \frac{dw_0}{dx} + \phi_x = \gamma_{xz}$$

دیده می‌شود که شیب در مرکز تیر صفر نیست (برخلاف تیر کلاسیک)

$$\left(I_y = bh^3/12 \right)$$

$$\mathrm{II} \Rightarrow \left.\frac{dw_0}{dx}\right|_{x=a/2} = \frac{F_0 b}{2kG_{xz}^b bh} \qquad (\mathrm{V})$$

اگر

$$\left.\frac{dw_0^b}{dx}\right|_{x=a/2} = -\phi_x\left(\frac{a}{2}\right) = 0 \qquad (\mathrm{VI})$$

با انتگرال‌گیری از رابطه II داریم

$$w_0(x) = \underbrace{\frac{F_0 b a^3}{48 E_x^b I_y}\left[3\left(\frac{x}{a}\right) - 4\left(\frac{x}{a}\right)^3\right]}_{w_0^b \equiv w_0^{CLPT}} + \frac{F_0 b a}{2kG_{xz}^b bh}\left(\frac{x}{a}\right) \qquad \mathrm{VII}$$

مقدار ماکزیمم خیز

$$\begin{aligned} w_{max} &= \frac{F_0 b a^3}{48 E_{xx}^b I_{yy}} + \frac{F_0 b a}{4 K G_{xz}^b bh} \\ &= \frac{F_0 b a^3}{48 E_{xx}^b I_{yy}}\left[1 + \left(\frac{E_{xx}^b}{K G_{xz}^b}\right)\left(\frac{h}{a}\right)^2\right] \end{aligned} \qquad \mathrm{VIII}$$

دیده می‌شود که اثر در نظر گرفتن تغییر شکل برشی، زیاد شدن خیز است. که مقدار آن به نسبت $\frac{E^b}{G^b}$

و نسبت مدول‌ها $\frac{h}{a}$ بستگی دارد. برای تیرهای بلند و نازک قابل صرفنظر کردن است.

6.3-3 Buckling

کافی است در معادله کلی تیر FSDT، ترم‌های اینرسی و بار عرضی را صفر قرار دهیم.

$$KG^b_{xz}bh\left(\frac{d^2W}{dx^2}+\frac{d\mathcal{X}}{dx}\right)+b\hat{N}_{xx}\frac{d^2W}{dx^2}=0 \qquad (6.3\text{-}19)$$

$$E^b_{xx}I_{yy}\frac{d^2\mathcal{X}}{dx^2}-KG^b_{xz}bh\left(\frac{dW}{dx}+\mathcal{X}\right)=0$$

FSDT کمانش تیر

$\mathcal{X} \equiv \phi_x$

از دو معادله فوق می‌توان یافت ($\hat{N}_x = -N_x^0$) :

$$K G^b_{xz} bh \frac{d\mathcal{X}}{dx} = -\left(KG^b_{xz}bh - bN_x^0\right)\frac{d^2w}{dx^2} \qquad (6.3\text{-}20)$$

$$\Rightarrow K G^b_{xz} bh\, \mathcal{X}(x) = -\left(KG^b_{xz}bh - bN_x^0\right)\frac{dw}{dx} + K_1 \qquad (6.3\text{-}21)$$

$$E^b_{xx} I_{yy}\left(1-\frac{bN^0_{xx}}{KG^b_{xz}bh}\right)\frac{d^4W}{dx^4}+bN^0_{xx}\frac{d^2W}{dx^2}=0 \qquad (6.3\_22)$$

که جواب این معادله

$$w(x) = C_1 \sin\lambda x + C_2 \cos\lambda x + C_3 x + C_4 \qquad (6.3-23)$$

که

$$\lambda^2=\frac{bN^0_{xx}}{\left(1-\frac{bN^0_{xx}}{KG^b_{xz}bh}\right)E^b_{xx}I_{yy}} \quad \text{or} \quad bN^0_{xx}=\frac{\lambda^2 E^b_{xx}I_{yy}}{\left(1+\frac{\lambda^2 E^b_{xx}I_{yy}}{KG^b_{xz}bh}\right)} \qquad (6.3-24)$$

(6.3-8)

مثال: تیر دو سر مفصل

$\{M\}=[D]\{\varepsilon^{(1)}\}$

$$W(0)=0,\quad W(a)=0,\quad \frac{d\mathcal{X}}{dx}(0)=0,\quad \frac{d\mathcal{X}}{dx}(a)=0 \tag{4.3.34a}$$

In view of Eq. (~~4.3.29~~ 6.3-20), the above conditions are equivalent to

$$W(0)=0,\quad W(a)=0,\quad \frac{d^2W}{dx^2}(0)=0,\quad \frac{d^2W}{dx^2}(a)=0 \tag{4.3.34b}$$

The boundary conditions in Eq. (4.3.34b) lead to the result $c_2=c_3=c_4=0$, and for $c_1 \neq 0$ the requirement

$$\sin\lambda a=0 \ \text{ implies } \ \lambda a=n\pi \tag{4.3.35}$$

Substituting for λ from Eq. (~~4.3.35~~ 6.3-24) into Eq. (4.3.33) for N_{xx}^0, we obtain

$$\begin{aligned} bN_{xx}^0 &= E_{xx}^b I_{yy}\left(\frac{n\pi}{a}\right)^2\left[\frac{KG_{xz}^b bh}{KG_{xz}^b bh+E_{xx}^b I_{yy}\left(\frac{n\pi}{a}\right)^2}\right] \\ &= E_{xx}^b I_{yy}\left(\frac{n\pi}{a}\right)^2\left[1-\frac{E_{xx}^b I_{yy}\left(\frac{n\pi}{a}\right)^2}{KG_{xz}^b bh+E_{xx}^b I_{yy}\left(\frac{n\pi}{a}\right)^2}\right] \end{aligned} \tag{4.3.36}$$

The critical buckling load is given by the minimum $(n=1)$

$$bN_{cr}=E_{xx}^b I_{yy}\left(\frac{\pi}{a}\right)^2\left[1-\frac{E_{xx}^b I_{yy}\left(\frac{\pi}{a}\right)^2}{KG_{xz}^b bh+E_{xx}^b I_{yy}\left(\frac{\pi}{a}\right)^2}\right] \tag{4.3.37}$$

6-3-4 Vibration

برای یافتن معادله ارتعاشی تیر کافیست در معادله کلی بارگذاری محوری و عرضی را صفر قرار دهیم و همچنین خیز را بصورت متناوب فرض کنیم . داریم:

$$KG^b_{xz}bh\left(\frac{d^2W}{dx^2}+\frac{d\mathcal{X}}{dx}\right)+\omega^2\hat{I}_0W=0 \qquad (6.3\text{-}25\,a)$$

$$E^b_{xx}I_{yy}\frac{d^2\mathcal{X}}{dx^2}-KG^b_{xz}bh\left(\frac{dW}{dx}+\mathcal{X}\right)+\omega^2\hat{I}_2\mathcal{X}=0 \qquad (6.3\text{-}25\,b)$$

FSDT ارتعاشی تیر

$\mathcal{X}\equiv\phi_x$

$u=u_0+z\mathcal{X}$

با روند مشابه می‌توان $\mathcal{X}$ را یافت:

$$KG^b_{xz}bh\frac{d\mathcal{X}}{dx}=-\hat{I}_0\omega^2W-KG^b_{xz}bh\frac{d^2W}{dx^2} \qquad (6.3\text{-}26)$$

$$\Rightarrow \quad E^b_{xx}I_{yy}\frac{d^4W}{dx^4}+\left(\frac{E^b_{xx}I_{yy}\hat{I}_0}{KG^b_{xz}bh}+\hat{I}_2\right)\omega^2\frac{d^2W}{dx^2}-\left(1-\frac{\omega^2\hat{I}_2}{KG^b_{xz}bh}\right)\hat{I}_0\omega^2W=0 \qquad (6.3\text{-}27)$$

or

$$p\frac{d^4W}{dx^4}+q\frac{d^2W}{dx^2}-rW=0 \qquad (6.3\text{-}28)$$

where

$$p=E^b_{xx}I_{yy},\quad q=\left(\frac{E^b_{xx}I_{yy}}{KG^b_{xz}bh}+\frac{\hat{I}_2}{\hat{I}_0}\right)\hat{I}_0\omega^2,\quad r=\left(1-\frac{\omega^2\hat{I}_2}{KG^b_{xz}bh}\right)\hat{I}_0\omega^2 \qquad (6.3\text{-}29)$$

جواب عمومی معادله فوق چنین خواهد بود

$$W(x) = C_1 \sin \lambda x + C_2 \cos \lambda x + C_3 \sinh(\mu x) + C_4 \cosh(\mu x) \qquad (6.3\text{-}30)$$

$$\lambda = \sqrt{\frac{1}{2p}\left(q+\sqrt{q^2+4pr}\right)} \quad , \quad \mu = \sqrt{\frac{1}{2p}\left(-q+\sqrt{q^2+4pr}\right)} \qquad (6.3\text{-}31)$$

توجه شود که

$$(2\lambda^2 p - q)^2 = q^2 + 4pr \quad \text{یا} \quad p\lambda^4 - q\lambda^2 - r = 0 \qquad (6.3\text{-}32)$$

اگر مقدار (6.3-30) را در (6.3-27) قرار دهیم داریم:

of ω as

$$P\omega^4 - Q\omega^2 + R = 0 \qquad (6.3\text{-}33)$$

where

$$P = \frac{\hat{I}_2}{KG^b_{xz}bh}\ , \quad Q = \left[1 + \left(\frac{E^b_{xx}I_{yy}}{KG^b_{xz}bh} + \frac{\hat{I}_2}{\hat{I}_0}\right)\lambda^2\right], \quad R = \left(\frac{E^b_{xx}I_{yy}}{\hat{I}_0}\right)\lambda^4 \qquad (6.3\text{-}34)$$

دو دسته جواب برای این معادله قابل محاسبه است.

$$(\omega^2)_1 = \frac{1}{2P}\left(Q - \sqrt{Q^2 - 4PR}\right), \quad (\omega^2)_2 = \frac{1}{2P}\left(Q + \sqrt{Q^2 - 4PR}\right) \qquad (6.3\text{-}35)$$

اگر $PQ>0$ باشد و $Q^2-4PR>0$ باشد مقدار اول کمتری مقدار فرکانس را خواهد داد.

اگر اینرسی دورانی قابل صرف نظر کردن باشد آنگاه ($P=0$) :

$$\omega^2 = \frac{R}{\bar{Q}} \quad \text{و} \quad \bar{Q} = \left[1 + \left(\frac{E_x^b I_y}{K G_{xz}^b bh}\right)\lambda^2\right] \quad \text{و} \quad R = \left(\frac{E_x^b I_y}{\hat{I}_0}\right)\lambda^4 \qquad (6.3\text{-}36)$$

مثال: تیر دو سر مفصل

با استفاده از شرایط مرزی می‌رسیم که $c_2=c_3=c_4=0$ و داریم

$$c_1 \sin \lambda a = 0, \text{ which implies } \lambda_n = \frac{n\pi}{a}$$

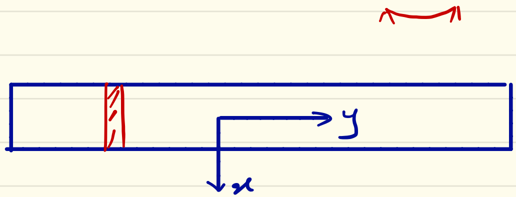
با قرار دادن در (6.3-36) و سپس قرار دادن در (6.3-35) در جواب بدست می‌آید که اولی کوچکتر است.

$$\omega_n = \left(\frac{n\pi}{a}\right)^2 \sqrt{\frac{E_{xx}^b I_{yy}}{\hat{I}_0}} \sqrt{1 - \frac{(\frac{n\pi}{a})^2 E_{xx}^b I_{yy}}{K G_{xz}^b bh + (\frac{n\pi}{a})^2 E_{xx}^b I_{yy}}}$$

6.4 Cylindrical Bending Using CLPT

6.4.1 Governing Equations

$q(x)$, $\dfrac{\partial}{\partial y} = 0$



$\Rightarrow u(x)$, $w_0(x)$

$$A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{16}\frac{\partial^2 v_0}{\partial x^2} - B_{11}\frac{\partial^3 w_0}{\partial x^3} - \frac{\partial N_{xx}^T}{\partial x} = I_0\frac{\partial^2 u_0}{\partial t^2} - I_1\frac{\partial^3 w_0}{\partial x \partial t^2} \quad (6.4\text{-}1a)$$

$$A_{16}\frac{\partial^2 u_0}{\partial x^2} + A_{66}\frac{\partial^2 v_0}{\partial x^2} - B_{16}\frac{\partial^3 w_0}{\partial x^3} - \frac{\partial N_{xy}^T}{\partial x} = I_0\frac{\partial^2 v_0}{\partial t^2} \quad (6.4\text{-}1b)$$

معادلات تعادل ورق استوانه ای CLPT

$$B_{11}\frac{\partial^3 u_0}{\partial x^3} + B_{16}\frac{\partial^3 v_0}{\partial x^3} - D_{11}\frac{\partial^4 w_0}{\partial x^4} + \frac{\partial}{\partial x}\left(\hat{N}_{xx}\frac{\partial w_0}{\partial x}\right) - \frac{\partial^2 M_{xx}^T}{\partial x^2} + q = I_0\frac{\partial^2 w_0}{\partial t^2} - I_2\frac{\partial^4 w_0}{\partial x^2 \partial t^2} + I_1\frac{\partial^3 u_0}{\partial x \partial t^2} \quad (6.4\text{-}1c)$$

که

$$(I_0, I_1, I_2) = \sum_{k=1}^{L}\int_{z_k}^{z_{k+1}} (1, z, z^2)\rho_0^{(k)}\, dz \quad (6.4\text{-}2)$$

در حالت کلی این سه معادله به هم کوپل هستند. اما برای cross-ply معادله دوم مستقل می شود

این معادلات را می توان چنین مرتب کرد:

$$A\frac{\partial^2 u_0}{\partial x^2} = B\frac{\partial^3 w_0}{\partial x^3} + A_{66}\frac{\partial N_{xx}^T}{\partial x} - A_{16}\frac{\partial N_{xy}^T}{\partial x} + A_{66}I_0\frac{\partial^2 u_0}{\partial t^2} - A_{16}I_0\frac{\partial^2 v_0}{\partial t^2} - A_{66}I_1\frac{\partial^3 w_0}{\partial x \partial t^2} \qquad (6.4\text{-}3a)$$

$$A\frac{\partial^2 v_0}{\partial x^2} = C\frac{\partial^3 w_0}{\partial x^3} + A_{11}\frac{\partial N_{xy}^T}{\partial x} - A_{16}\frac{\partial N_{xx}^T}{\partial x} + A_{11}I_0\frac{\partial^2 v_0}{\partial t^2} - A_{16}I_0\frac{\partial^2 u_0}{\partial t^2} + A_{16}I_1\frac{\partial^3 w_0}{\partial x \partial t^2} \qquad (6.4\text{-}3b)$$

$$D\frac{\partial^4 w_0}{\partial x^4} = \bar{B}\frac{\partial^2 N_{xx}^T}{\partial x^2} + \bar{C}\frac{\partial^2 N_{xy}^T}{\partial x^2} - (I_1 - \bar{B}I_0)\frac{\partial^3 u_0}{\partial x \partial t^2} + \bar{C}I_0\frac{\partial^3 v_0}{\partial x \partial t^2} - I_0\frac{\partial^2 w_0}{\partial t^2} + (I_2 - \bar{B}I_1)\frac{\partial^4 w_0}{\partial x^2 \partial t^2} - \frac{\partial^2 M_{xx}^T}{\partial x^2} + \frac{\partial}{\partial x}\left(\hat{N}_{xx}\frac{\partial w_0}{\partial x}\right) + q \qquad (6.4\text{-}3c)$$

where

$$A = A_{11}A_{66} - A_{16}A_{16}, \quad B = B_{11}A_{66} - B_{16}A_{16}, \quad C = A_{11}B_{16} - A_{16}B_{11} \qquad (6.4\text{-}4)$$

$$D = D_{11} - B_{11}\bar{B} - B_{16}\bar{C}, \quad \bar{B} = \frac{B}{A}, \quad \bar{C} = \frac{C}{A}$$

برای cross-ply ، $C=0$ می شود.

6-4-2 Bending

$$
\begin{aligned}
A\frac{d^2u_0}{dx^2} &= B\frac{d^3w_0}{dx^3} + A_{66}\frac{dN_{xx}^T}{dx} - A_{16}\frac{dN_{xy}^T}{dx}\\
A\frac{d^2v_0}{dx^2} &= C\frac{d^3w_0}{dx^3} + A_{11}\frac{dN_{xy}^T}{dx} - A_{16}\frac{dN_{xx}^T}{dx}\\
D\frac{d^4w_0}{dx^4} &= \bar{B}\frac{d^2N_{xx}^T}{dx^2} + \bar{C}\frac{d^2N_{xy}^T}{dx^2} - \frac{d^2M_{xx}^T}{dx^2} + q
\end{aligned}
\qquad (6.4\text{-}5)
$$

معادله سوم مستقل از دو معادله دیگر است.

برای حالتی که دو لبه در جهت x هردو ساده یا گیردار باشند جواب دقیق برای حل این معادلات وجود دارد.

چون معادله سوم مستقل است از آن انتگرال می‌گیریم و در دو معادله دیگر قرار می‌دهیم.

$$
\begin{aligned}
D\frac{d^3w_0}{dx^3} &= \bar{B}\frac{dN_{xx}^T}{dx} + \bar{C}\frac{dN_{xy}^T}{dx} - \frac{dM_{xx}^T}{dx} + \int_0^x q(\xi)\ d\xi + c_1\\
A\frac{d^2u_0}{dx^2} &= \hat{B}\int_0^x q(\xi)\ d\xi + G_1\frac{dN_{xx}^T}{dx} + F_1\frac{dN_{xy}^T}{dx} - \hat{B}\frac{dM_{xx}^T}{dx} + a_1\\
A\frac{d^2v_0}{dx^2} &= \hat{C}\int_0^x q(\xi)\ d\xi + G_2\frac{dN_{xx}^T}{dx} + F_2\frac{dN_{xy}^T}{dx} - \hat{C}\frac{dM_{xx}^T}{dx} + b_1
\end{aligned}
\qquad (6.4\text{-}6)
$$

که در آن

$$
\begin{aligned}
G_1 &= \frac{\bar{B}B}{D} + A_{66},\quad F_1 = \frac{\bar{B}B}{D} - A_{16},\quad \hat{B} = \frac{B}{D}\\
G_2 &= \frac{\bar{B}C}{D} - A_{16},\quad F_2 = \frac{\bar{B}C}{D} + A_{11},\quad \hat{C} = \frac{C}{D}\\
a_1 &= \hat{B}c_1,\quad b_1 = \hat{C}c_1
\end{aligned}
\qquad (6.4\text{-}7)
$$

وسی

$$Au_0(x) = \hat{B}\int_0^x \left[\int_0^\xi \left(\int_0^\eta q(\zeta)\ d\zeta\right)\ d\eta\right] d\xi + G_1\int_0^x N_{xx}^T(\xi)d\xi + F_1\int_0^x N_{xy}^T(\xi)d\xi$$
$$- \hat{B}\int_0^x M_{xx}^T(\xi)d\xi + a_1\frac{x^2}{2} + a_2 x + a_3$$

$$Av_0(x) = \hat{C}\int_0^x \left[\int_0^\xi \left(\int_0^\eta q(\zeta)\ d\zeta\right)\ d\eta\right] d\xi + G_2\int_0^x N_{xx}^T(\xi)d\xi + F_2\int_0^x N_{xy}^T(\xi)d\xi$$
$$- \hat{C}\int_0^x M_{xx}^T(\xi)d\xi + b_1\frac{x^2}{2} + b_2 x + b_3$$

(6.4-8)

$$D\frac{dw_0}{dx} = \int_0^x \left[\int_0^\xi \left(\int_0^\eta q(\zeta)\ d\zeta\right)\ d\eta\right] d\xi + \bar{B}\int_0^x N_{xx}^T(\xi)d\xi + \bar{C}\int_0^x N_{xy}^T(\xi)d\xi$$
$$- \int_0^x M_{xx}^T(\xi)d\xi + c_1\frac{x^2}{2} + c_2 x + c_3$$

$$Dw_0(x) = \int_0^x \left\{\int_0^\xi \left[\int_0^\eta \left(\int_0^\zeta q(\mu)d\mu\right)\ d\zeta\right]\ d\eta\right\} d\xi + \bar{B}\int_0^x \left(\int_0^\xi N_{xx}^T(\eta)d\eta\right) d\xi$$
$$+ \bar{C}\int_0^x \left(\int_0^\xi N_{xy}^T(\eta)d\eta\right) d\xi - \int_0^x \left(\int_0^\xi M_{xx}^T(\eta)d\eta\right) d\xi$$
$$+ c_1\frac{x^3}{6} + c_2\frac{x^2}{2} + c_3 x + c_4$$