جلسه ۲۰ مواد مرکب بسم الله الرحمن الرحیم

کرنش های غیرخطی

$$\varepsilon_x = \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 + z\frac{\partial \phi_x}{\partial x}$$ خطی برحسب z

$$\varepsilon_y = \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 + z\frac{\partial \phi_y}{\partial y}$$ 〃 〃

$$\gamma_{xy} = \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right) + z\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right)$$ خطی برحسب z

(5.2-41)

$$\gamma_{xz} = \frac{\partial w_0}{\partial x} + \phi_x$$

$$\gamma_{yz} = \frac{\partial w_0}{\partial y} + \phi_y$$

ثابت در امتداد z
(تقریب)

$$\varepsilon_z = 0$$

(5.2-5)

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{yz}^0 \\ \gamma_{xz}^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} \varepsilon_x^{(1)} \\ \varepsilon_y^{(1)} \\ \gamma_{yz}^{(1)} \\ \gamma_{xz}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \\ \frac{\partial w_0}{\partial x} + \phi_x \\ \frac{\partial w_0}{\partial y} + \phi_y \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \end{Bmatrix} + z\begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ 0 \\ 0 \\ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \end{Bmatrix}$$

5.2.2 Equation of Motion

$$0 = \int_0^T (\delta U + \delta V - \delta K)\,dt \qquad (5.2\text{-}6)$$

$$\delta U = \int_{\Omega_0} \left\{ \int_{-h/2}^{h/2} \Big[\sigma_x \left(\delta \varepsilon_x^0 + z\,\delta \varepsilon_x^{(1)}\right) + \sigma_y \left(\delta \varepsilon_y^0 + z\,\delta \varepsilon_y^{(1)}\right) + \sigma_{xy} \left(\delta \gamma_{xy}^0 + z\,\delta \gamma_{xy}^{(1)}\right) + \sigma_{xz}\,\delta \gamma_{xz}^0 + \sigma_{yz}\,\delta \gamma_{yz}^0 \Big] dz \right\} dx\,dy$$

$$\delta V = -\int_{\Omega_0} \big[(q_b + q_t)\,\delta w_0 \big]\,dx\,dy - \int_{\Gamma_\sigma} \int_{-h/2}^{h/2} \Big[\hat{\sigma}_n \left(\delta u_n + z\,\delta \phi_n\right) + \hat{\sigma}_{ns} \left(\delta u_s + z\,\delta \phi_s\right) + \hat{\sigma}_{nz}\,\delta w_0 \Big] dz\,ds$$

$$\delta K = \int_{\Omega_0} \int_{-h/2}^{h/2} \rho_0 \Big[\left(\dot{u}_0 + z\dot{\phi}_x\right)\left(\delta \dot{u}_0 + z\,\delta \dot{\phi}_x\right) + \left(\dot{v}_0 + z\dot{\phi}_y\right)\left(\delta \dot{v}_0 + z\,\delta \dot{\phi}_y\right) + \dot{w}_0\,\delta \dot{w}_0 \Big] dz\,dx\,dy$$

$$(5.2\text{-}7)$$

با جایگزاری (5.2-7) در رابطه (5.2-6) و انتگرال گیری در جهت z داریم:

$$
\begin{aligned}
0 = \int_0^T \Bigg\{ \int_{\Omega_0} \Big[& N_{xx}\delta\varepsilon_{xx}^{(0)} + M_{xx}\delta\varepsilon_{xx}^{(1)} + N_{yy}\delta\varepsilon_{yy}^{(0)} + M_{yy}\delta\varepsilon_{yy}^{(1)} + N_{xy}\delta\gamma_{xy}^{(0)} + M_{xy}\delta\gamma_{xy}^{(1)} \\
& + Q_x\delta\gamma_{xz}^{(0)} + Q_y\delta\gamma_{yz}^{(0)} - q\delta w_0 - I_0\left(\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + \dot{w}_0\delta\dot{w}_0\right) \\
& - I_1\left(\dot{\phi}_x\delta\dot{u}_0 + \dot{\phi}_y\delta\dot{v}_0 + \delta\dot{\phi}_x\dot{u}_0 + \delta\dot{\phi}_y\dot{v}_0\right) - I_2\left(\dot{\phi}_x\delta\dot{\phi}_x + \dot{\phi}_y\delta\dot{\phi}_y\right)\Big]\, dxdy \\
& - \int_{\Gamma_\sigma}\left(\hat{N}_{nn}\delta u_n + \hat{N}_{ns}\delta u_s + \hat{M}_{nn}\delta\phi_n + \hat{M}_{ns}\delta\phi_s + \hat{Q}_n\delta w_0\right) ds \Bigg\} dt \qquad (3.4.9)
\end{aligned}
$$

(5.2-8)

تنش برشی در تئوری FSDT و فاکتور اصلاح تنش برشی

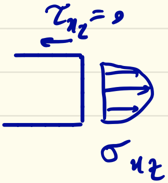
بصورت عادی داشتیم:

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} dz \qquad (5.2-9)$$

یعنی در این تئوری مقدار تنش برشی ثابت در نظر گرفته می شود که برابر است با

$$\sigma_{xz} = \frac{Q_x}{A} \qquad (5.2-10)$$

A ← سطح مقطع



اما می دانیم که در واقعیت این تنش توزیع درجه دوم دارد. (حداقل)

این فرض ثابت بودن، باعث می‌شود انرژی کرنشی محاسبه شده دارای خطایی نسبت به واقعیت باشد.

این خطا را با ضریبی اصلاح می‌توان کرد.

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = K \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} dz \qquad (5.2\text{-}11)$$

$$K = \frac{U^C}{U^F} \qquad (5.2\text{-}12)$$

U^C ← انرژی کرنشی با استفاده از توزیع واقعی

U^F ← انرژی کرنشی با استفاده از فرض ثابت بودن Q_x

مثلاً برای سطح مقطع مستطیلی شکل توزیع واقعی

$$\sigma_{xz}^C = \frac{3Q}{2bh}\left[1 - \left(\frac{2z}{h}\right)^2\right] \qquad (5.2\text{-}13)$$

$$\Rightarrow \begin{cases} U^C = \dfrac{1}{2G_{13}} \displaystyle\int_A (\sigma_{xz}^C)^2 \, dA = \dfrac{3Q^2}{5G_{13}bh} \\ U^F = \dfrac{1}{2G_{13}} \displaystyle\int_A (\sigma_{xz}^F)^2 \, dA = \dfrac{Q^2}{3G_{13}bh} \end{cases} \qquad (5.2\text{-}14)$$

$$K = \frac{U^c}{U^F} = \frac{5}{6}$$ برای مقطع مستطیلی

با مرتب کردن رابطه (5.2-8) داریم:

$$
\begin{aligned}
0 = \int_0^T \int_{\Omega_0} \Big[& -\left(N_{xx,x} + N_{xy,y} - I_0 \ddot{u}_0 - I_1 \ddot{\phi}_x\right) \delta u_0 \\
& -\left(N_{xy,x} + N_{yy,y} - I_0 \ddot{v}_0 - I_1 \ddot{\phi}_y\right) \delta v_0 \\
& -\left(M_{xx,x} + M_{xy,y} - Q_x - I_2 \ddot{\phi}_x - I_1 \ddot{u}_0\right) \delta \phi_x \\
& -\left(M_{xy,x} + M_{yy,y} - Q_y - I_2 \ddot{\phi}_y - I_1 \ddot{v}_0\right) \delta \phi_y \\
& -\left(Q_{x,x} + Q_{y,y} + \mathcal{N}(w_0) + q - I_0 \ddot{w}_0\right) \delta w_0 \Big] dx dy \\
+ \int_0^T \int_\Gamma \Big[& \left(N_{nn} - \hat{N}_{nn}\right) \delta u_n + \left(N_{ns} - \hat{N}_{ns}\right) \delta u_s + \left(Q_n - \hat{Q}_n\right) \delta w_0 \\
& + \left(M_{nn} - \hat{M}_{nn}\right) \delta \phi_n + \left(M_{ns} - \hat{M}_{ns}\right) \delta \phi_s \Big] ds dt
\end{aligned}
$$

(5.2-15)

که در آن

$$\mathcal{N}(w_0) = \frac{\partial}{\partial x}\left(N_{xx}\frac{\partial w_0}{\partial x} + N_{xy}\frac{\partial w_0}{\partial y}\right) + \frac{\partial}{\partial y}\left(N_{xy}\frac{\partial w_0}{\partial x} + N_{yy}\frac{\partial w_0}{\partial y}\right)$$

$$\mathcal{P}(w_0) = \left(N_{xx}\frac{\partial w_0}{\partial x} + N_{xy}\frac{\partial w_0}{\partial y}\right) n_x + \left(N_{xy}\frac{\partial w_0}{\partial x} + N_{yy}\frac{\partial w_0}{\partial y}\right) n_y$$

(5.2-16)

حال گوئیم:

$$\delta u_0: \quad \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \frac{\partial^2 u_0}{\partial t^2} + I_1 \frac{\partial^2 \phi_x}{\partial t^2}$$

$$\delta v_0: \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = I_0 \frac{\partial^2 v_0}{\partial t^2} + I_1 \frac{\partial^2 \phi_y}{\partial t^2}$$

$$\delta w_0: \quad \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \mathcal{N}(w_0) + q = I_0 \frac{\partial^2 w_0}{\partial t^2}$$

$$\delta \phi_x: \quad \frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = I_2 \frac{\partial^2 \phi_x}{\partial t^2} + I_1 \frac{\partial^2 u_0}{\partial t^2}$$

$$\delta \phi_y: \quad \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - Q_y = I_2 \frac{\partial^2 \phi_y}{\partial t^2} + I_1 \frac{\partial^2 v_0}{\partial t^2}$$

(5.2-17)

معادلات حاکم بر ورق FSDT

متغیرهای اولیه $u_n, u_s, w_0, \phi_n, \phi_s$ (5.2-18)

متغیرهای ثانویه $N_n, N_{ns}, Q_n, M_n, M_{ns}$

لذا شرایط مرزی طبیعی (انرژی) natural B.C.

$$N_n - \hat{N}_n = 0, \quad N_{ns} - \hat{N}_{ns} = 0, \quad Q_n - \hat{Q}_n = 0, \quad M_n - \hat{M}_n = 0, \quad M_{ns} - \hat{M}_{ns} = 0$$

(5.2-19)

که در آن

$$Q_n = Q_x n_x + Q_y n_y + \mathcal{P}(w_0)$$

5-2-3 Laminate Constitutive Equation

قبلاً دیدیم:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{Bmatrix}_k = [\bar{Q}]_k \left(\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} - \begin{Bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{Bmatrix}\Delta T \right) - \begin{bmatrix}0 & 0 & \bar{e}_{31}\\ 0 & 0 & \bar{e}_{32}\\ 0 & 0 & \bar{e}_{36}\end{bmatrix}_k \begin{Bmatrix}E_x\\ E_y\\ E_z\end{Bmatrix}_k \qquad (5.2\text{-}20)$$

با انتگرال گیری می رسیم به:

$$\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\end{Bmatrix} = [A] \begin{Bmatrix}\varepsilon_x^{0}\\ \varepsilon_y^{0}\\ \gamma_{xy}^{0}\end{Bmatrix} + [B] \begin{Bmatrix}\varepsilon_x^{(1)}\\ \varepsilon_y^{(1)}\\ \gamma_{xy}^{(1)}\end{Bmatrix} \qquad (5.2\text{-}21)$$

$$\begin{Bmatrix}M_x\\ M_y\\ M_{xy}\end{Bmatrix} = [B] \begin{Bmatrix}\varepsilon_x^{0}\\ \varepsilon_y^{0}\\ \gamma_{xy}^{0}\end{Bmatrix} + [D] \begin{Bmatrix}\varepsilon_x^{(1)}\\ \varepsilon_y^{(1)}\\ \gamma_{xy}^{(1)}\end{Bmatrix} \qquad (5.2\text{-}22)$$

در تئوری FSDT ماتریس های A, B, D تغییری نکنند، فقط تعریف K
تغییر خواهد کرد.

$$
\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \end{Bmatrix}
+ \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \end{Bmatrix}
$$

در (5.2-23)

$$
\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \end{Bmatrix}
+ \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \end{Bmatrix}
$$

معادلات فضای حالت در تئوری FSDT

$$
\begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = K \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \frac{\partial w_0}{\partial y} + \phi_y \\ \frac{\partial w_0}{\partial x} + \phi_x \end{Bmatrix}
$$

5-2-4 Equations of Motion in Terms of Displacements

با استفاده از روابط (5.2-23) می‌توان روابط حرکت (5.2-17) را برحسب جابجایی‌های عمومی
ϕ_x و ϕ_y و w_0 و v_0 و u_0 می‌توان چنین نوشت (با درنظرگرفتن دما و خاصیت پیزوالکتریک)

$$
\begin{aligned}
&A_{11}\left(\frac{\partial^2 u_0}{\partial x^2}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2}\right)+A_{12}\left(\frac{\partial^2 v_0}{\partial y\partial x}+\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y\partial x}\right)+\\
&A_{16}\left(\frac{\partial^2 u_0}{\partial y\partial x}+\frac{\partial^2 v_0}{\partial x^2}+\frac{\partial^2 w_0}{\partial x^2}\frac{\partial w_0}{\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y\partial x}\right)+\\
&B_{11}\frac{\partial^2 \phi_x}{\partial x^2}+B_{12}\frac{\partial^2 \phi_y}{\partial y\partial x}+B_{16}\left(\frac{\partial^2 \phi_x}{\partial x\partial y}+\frac{\partial^2 \phi_y}{\partial x^2}\right)+\\
&A_{16}\left(\frac{\partial^2 u_0}{\partial x\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x\partial y}\right)+A_{26}\left(\frac{\partial^2 v_0}{\partial y^2}+\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y^2}\right)+\\
&A_{66}\left(\frac{\partial^2 u_0}{\partial y^2}+\frac{\partial^2 v_0}{\partial x\partial y}+\frac{\partial^2 w_0}{\partial x\partial y}\frac{\partial w_0}{\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2}\right)+\\
&B_{16}\frac{\partial^2 \phi_x}{\partial x\partial y}+B_{26}\frac{\partial^2 \phi_y}{\partial y^2}+B_{66}\left(\frac{\partial^2 \phi_x}{\partial y^2}+\frac{\partial^2 \phi_y}{\partial y\partial x}\right)-\\
&\left(\frac{\partial N_{xx}^T}{\partial x}+\frac{\partial N_{xy}^T}{\partial y}\right)-\left(\frac{\partial N_{xx}^P}{\partial x}+\frac{\partial N_{xy}^P}{\partial y}\right)=I_0\frac{\partial^2 u_0}{\partial t^2}+I_1\frac{\partial^2 \phi_x}{\partial t^2}
\end{aligned}
\qquad (5.2\text{-}24a)
$$

$$A_{16}\left(\frac{\partial^2 u_0}{\partial x^2}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2}\right)+A_{26}\left(\frac{\partial^2 v_0}{\partial y\partial x}+\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y\partial x}\right)+$$

$$A_{66}\left(\frac{\partial^2 u_0}{\partial y\partial x}+\frac{\partial^2 v_0}{\partial x^2}+\frac{\partial^2 w_0}{\partial x^2}\frac{\partial w_0}{\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y\partial x}\right)+$$

$$B_{16}\frac{\partial^2 \phi_x}{\partial x^2}+B_{26}\frac{\partial^2 \phi_y}{\partial y\partial x}+B_{66}\left(\frac{\partial^2 \phi_x}{\partial x\partial y}+\frac{\partial^2 \phi_y}{\partial x^2}\right)+$$

$$A_{12}\left(\frac{\partial^2 u_0}{\partial x\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x\partial y}\right)+A_{22}\left(\frac{\partial^2 v_0}{\partial y^2}+\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y^2}\right)+$$

$$A_{26}\left(\frac{\partial^2 u_0}{\partial y^2}+\frac{\partial^2 v_0}{\partial x\partial y}+\frac{\partial^2 w_0}{\partial x\partial y}\frac{\partial w_0}{\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2}\right)+$$

$$B_{12}\frac{\partial^2 \phi_x}{\partial x\partial y}+B_{22}\frac{\partial^2 \phi_y}{\partial y^2}+B_{26}\left(\frac{\partial^2 \phi_x}{\partial y^2}+\frac{\partial^2 \phi_y}{\partial x\partial y}\right)-$$

$$\left(\frac{\partial N_{xy}^T}{\partial x}+\frac{\partial N_{yy}^T}{\partial y}\right)-\left(\frac{\partial N_{xy}^P}{\partial x}+\frac{\partial N_{yy}^P}{\partial y}\right)=I_0\frac{\partial^2 v_0}{\partial t^2}+I_1\frac{\partial^2 \phi_y}{\partial t^2}$$

(5.2-24b)

$$KA_{55}\left(\frac{\partial^2 w_0}{\partial x^2}+\frac{\partial \phi_x}{\partial x}\right)+KA_{45}\left(\frac{\partial^2 w_0}{\partial y\partial x}+\frac{\partial \phi_y}{\partial x}\right)+$$

$$KA_{45}\left(\frac{\partial^2 w_0}{\partial x\partial y}+\frac{\partial \phi_x}{\partial y}\right)+KA_{44}\left(\frac{\partial^2 w_0}{\partial y^2}+\frac{\partial \phi_y}{\partial y}\right)+$$

$$\mathcal{N}(w)+q-\left(\frac{\partial Q_x^P}{\partial x}+\frac{\partial Q_y^P}{\partial y}\right)=I_0\frac{\partial^2 w_0}{\partial t^2}$$

(5.2-24c)

$$
\begin{aligned}
&B_{11}\left(\frac{\partial^2 u_0}{\partial x^2}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2}\right)+B_{12}\left(\frac{\partial^2 v_0}{\partial y\partial x}+\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y\partial x}\right)+\\
&B_{16}\left(\frac{\partial^2 u_0}{\partial y\partial x}+\frac{\partial^2 v_0}{\partial x^2}+\frac{\partial^2 w_0}{\partial x^2}\frac{\partial w_0}{\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y\partial x}\right)+\\
&D_{11}\frac{\partial^2\phi_x}{\partial x^2}+D_{12}\frac{\partial^2\phi_y}{\partial y\partial x}+D_{16}\left(\frac{\partial^2\phi_x}{\partial x\partial y}+\frac{\partial^2\phi_y}{\partial x^2}\right)+\\
&B_{16}\left(\frac{\partial^2 u_0}{\partial x\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x\partial y}\right)+B_{26}\left(\frac{\partial^2 v_0}{\partial y^2}+\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y^2}\right)+\\
&B_{66}\left(\frac{\partial^2 u_0}{\partial y^2}+\frac{\partial^2 v_0}{\partial x\partial y}+\frac{\partial^2 w_0}{\partial x\partial y}\frac{\partial w_0}{\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2}\right)+\\
&D_{16}\frac{\partial^2\phi_x}{\partial x\partial y}+D_{26}\frac{\partial^2\phi_y}{\partial y^2}+D_{66}\left(\frac{\partial^2\phi_x}{\partial y^2}+\frac{\partial^2\phi_y}{\partial y\partial x}\right)-\\
&KA_{55}\left(\frac{\partial w_0}{\partial x}+\phi_x\right)-KA_{45}\left(\frac{\partial w_0}{\partial y}+\phi_y\right)-\\
&\left(\frac{\partial M_{xx}^T}{\partial x}+\frac{\partial M_{xy}^T}{\partial y}\right)-\left(\frac{\partial M_{xx}^P}{\partial x}+\frac{\partial M_{xy}^P}{\partial y}-Q_x^P\right)\\
&=I_2\frac{\partial^2\phi_x}{\partial t^2}+I_1\frac{\partial^2 u_0}{\partial t^2}
\end{aligned}
$$

(5.2-24d)

$$
\begin{aligned}
&B_{16}\left(\frac{\partial^2 u_0}{\partial x^2}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2}\right)+B_{26}\left(\frac{\partial^2 v_0}{\partial y\partial x}+\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y\partial x}\right)+\\
&B_{66}\left(\frac{\partial^2 u_0}{\partial y\partial x}+\frac{\partial^2 v_0}{\partial x^2}+\frac{\partial^2 w_0}{\partial x^2}\frac{\partial w_0}{\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y\partial x}\right)+\\
&D_{16}\frac{\partial^2 \phi_x}{\partial x^2}+D_{26}\frac{\partial^2 \phi_y}{\partial y\partial x}+D_{66}\left(\frac{\partial^2 \phi_x}{\partial x\partial y}+\frac{\partial^2 \phi_y}{\partial x^2}\right)+\\
&B_{12}\left(\frac{\partial^2 u_0}{\partial x\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x\partial y}\right)+B_{22}\left(\frac{\partial^2 v_0}{\partial y^2}+\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y^2}\right)+\\
&B_{26}\left(\frac{\partial^2 u_0}{\partial y^2}+\frac{\partial^2 v_0}{\partial x\partial y}+\frac{\partial^2 w_0}{\partial x\partial y}\frac{\partial w_0}{\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2}\right)+\\
&D_{12}\frac{\partial^2 \phi_x}{\partial x\partial y}+D_{22}\frac{\partial^2 \phi_y}{\partial y^2}+D_{26}\left(\frac{\partial^2 \phi_x}{\partial y^2}+\frac{\partial^2 \phi_y}{\partial x\partial y}\right)-\\
&KA_{45}\left(\frac{\partial w_0}{\partial x}+\phi_x\right)-KA_{44}\left(\frac{\partial w_0}{\partial y}+\phi_y\right)-\\
&\left(\frac{\partial M_{xy}^T}{\partial x}+\frac{\partial M_{yy}^T}{\partial y}\right)-\left(\frac{\partial M_{xy}^P}{\partial x}+\frac{\partial M_{yy}^P}{\partial y}-Q_y^P\right)\\
&=I_2\frac{\partial^2 \phi_y}{\partial t^2}+I_1\frac{\partial^2 v_0}{\partial t^2}
\end{aligned}
\qquad (5.2\text{-}24e)
$$

مثال : خمش استوانه‌ای (cylindrical bending) در تئوری FSDT چنین فرموله می‌شود

$$\frac{\partial}{\partial y} = 0$$

$$A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{16}\frac{\partial^2 v_0}{\partial x^2} + B_{11}\frac{\partial^2 \phi_x}{\partial x^2} + B_{16}\frac{\partial^2 \phi_y}{\partial x^2} - \frac{\partial N_{xx}^T}{\partial x} - \frac{\partial N_{xx}^P}{\partial x} = I_0\frac{\partial^2 u_0}{\partial t^2} + I_1\frac{\partial^2 \phi_x}{\partial t^2} \quad (a)$$

$$A_{16}\frac{\partial^2 u_0}{\partial x^2} + A_{66}\frac{\partial^2 v_0}{\partial x^2} + B_{16}\frac{\partial^2 \phi_x}{\partial x^2} + B_{66}\frac{\partial^2 \phi_y}{\partial x^2} - \frac{\partial N_{xy}^T}{\partial x} - \frac{\partial N_{xy}^P}{\partial x} = I_0\frac{\partial^2 v_0}{\partial t^2} + I_1\frac{\partial^2 \phi_y}{\partial t^2} \quad (b)$$

$$B_{11}\frac{\partial^2 u_0}{\partial x^2} + B_{16}\frac{\partial^2 v_0}{\partial x^2} + D_{11}\frac{\partial^2 \phi_x}{\partial x^2} + D_{16}\frac{\partial^2 \phi_y}{\partial x^2} - KA_{55}\left(\frac{\partial w_0}{\partial x} + \phi_x\right) - KA_{45}\phi_y - \frac{\partial M_{xx}^T}{\partial x} - \frac{\partial M_{xx}^P}{\partial x} + Q_x^P = I_2\frac{\partial^2 \phi_x}{\partial t^2} + I_1\frac{\partial^2 u_0}{\partial t^2} \quad (c)$$

$$B_{16}\frac{\partial^2 u_0}{\partial x^2} + B_{66}\frac{\partial^2 v_0}{\partial x^2} + D_{16}\frac{\partial^2 \phi_x}{\partial x^2} + D_{66}\frac{\partial^2 \phi_y}{\partial x^2} - KA_{44}\phi_y - KA_{45}\left(\frac{\partial w_0}{\partial x} + \phi_x\right) - \frac{\partial M_{xy}^T}{\partial x} - \frac{\partial M_{xy}^P}{\partial x} + Q_y^P = I_2\frac{\partial^2 \phi_y}{\partial t^2} + I_1\frac{\partial^2 v_0}{\partial t^2} \quad (d)$$

$$KA_{55}\left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial \phi_x}{\partial x}\right) + KA_{45}\frac{\partial \phi_y}{\partial x} + \frac{\partial}{\partial x}\left(N_{xx}\frac{\partial w_0}{\partial x}\right) + q - \frac{\partial Q_x^P}{\partial x} = I_0\frac{\partial^2 w_0}{\partial t^2} \quad (e)$$