بسم الله الرحمن الرحیم

مواد مرکب

جلسه ۱۹

با توجه به عبارت مرزی در رابطه (5.1-19) می‌توان گفت:

متغیرهای اولیه: $u_n , u_s , w_0 , \frac{\partial w_0}{\partial n} , \frac{\partial w_0}{\partial s}$

متغیرهای ثانویه: $N_n , N_{ns} , Q_n , M_n , M_{ns}$ (5.1-22)

شرایط مرزی طبیعی

natural b.c.

$$N_n - \hat{N}_n = 0 , \quad N_{ns} - \hat{N}_{ns} = 0 , \quad Q_n - \hat{Q}_n = 0$$

$$M_n - \hat{M}_n = 0 , \quad M_{ns} - \hat{M}_{ns} = 0 \qquad (5.1\text{-}23)$$

$$Q_n = \left(M_{x,x} + M_{xy,y} - I_1 \ddot{u}_0 + I_2 \frac{\partial \ddot{w}_0}{\partial x}\right) n_x + \left(M_{y,y} + M_{xy,x} - I_1 \ddot{v}_0 + I_2 \frac{\partial \ddot{w}_0}{\partial y}\right) n_y + P(w_0) \qquad (5.1\text{-}24)$$

اگر معادله (5.1-21) را برحسب جابجایی‌ها بنویسیم دیده می‌شود که u_0 و v_0 درجه 2 مشتق را دارند و w_0 هم درجه 4 مشتق را دارد. پس در کل ۸ ثابت انتگرال بیشتر نداریم. (به همین خاطر به تئوری CLPT تئوری درجه ۸ می‌گویند). لذا نمی‌توانیم بیش از ۸ شرایط مرزی داشته باشیم. اما رابطه (5.1-22) نشان می‌دهد که ۱۰ شرط مرزی در مجموع می‌توانیم داشته باشیم.

برای حل این مشکل در عبارت (5.1-19)، عبارت انتگرالی زیر را با تکنیک انتگرال جزء به جزء تغییر می‌دهیم.

$$-\oint_{\Gamma} M_{ns}\frac{\partial \delta w_0}{\partial s}\,ds = \oint_{\Gamma} \frac{\partial M_{ns}}{\partial s}\,\delta w_0\,ds - \left[M_{ns}\,\delta w_0\right]_{\Gamma} \qquad (5.1\text{-}25)$$

عبارت داخل براکت صفر است چون برروی نقاط ابتدا و انتهای یک بازه بسته نوشته شده است.

اما مقدار انتگرالی به ضریب δw_0 در رابطه (5.1-19) اضافه می‌شود پس می‌توان گفت:

$$V_n = Q_n + \frac{\partial M_{ns}}{\partial s} = \hat{Q}_n \qquad (5.1\text{-}26)$$

شرایط لبه آزاد کریشهف

پس متغیرهای ما چنین خواهند شد

$u_n , u_s , w_o , \frac{\partial w_o}{\partial n}$ متغیرهای اولیه (essential b.c.)

N_n , N_{ns} , V_n , M_n متغیرهای ثانویه (natural b.c.)

(5.1-27)

5_1_4_ Laminate Constitutive Equation

$$(5.1\text{-}10) \Rightarrow \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = [A] \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + [B] \begin{Bmatrix} \varepsilon_x^{(1)} \\ \varepsilon_y^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} \quad (5.1\text{-}28)$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = [B] \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + [D] \begin{Bmatrix} \varepsilon_x^{(1)} \\ \varepsilon_y^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} \quad (5.1\text{-}29)$$

$$\boxed{\begin{Bmatrix} \bar{N} \\ \bar{M} \end{Bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline B & D \end{array}\right] \begin{Bmatrix} \varepsilon^0 \\ \varepsilon^{(1)} \end{Bmatrix}} \quad (5.1\text{-}30)$$

$$\begin{Bmatrix} \bar{N} \\ \bar{m} \end{Bmatrix} = \begin{Bmatrix} N \\ m \end{Bmatrix} + \begin{Bmatrix} N^T \\ m^T \end{Bmatrix} + \begin{Bmatrix} N^P \\ m^P \end{Bmatrix} \qquad (5.1\text{-}31)$$

$$\{N^T\} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} [\bar{Q}]_k \{\bar{\alpha}\}_k \Delta T \, dz \qquad (5.1\text{-}32)$$

$$\{m^T\} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} [\bar{Q}]_k \{\bar{\alpha}\}_k \Delta T \, z \, dz$$

$$\{N^P\} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} [\bar{e}]_k \{E\}_k \, dz \qquad (5.1\text{-}33)$$

باقیمانده بیزوالکتریک

$$\{M^P\} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} [\bar{e}]_k \{E\}_k \, z \, dz$$

5.1-5 Equations of Motions in Terms of Displacements

داریم:

$$\{N\} = [A] \begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \end{Bmatrix} - [B] \begin{Bmatrix} \frac{\partial^2 w_0}{\partial x^2} \\ \frac{\partial^2 w_0}{\partial y^2} \\ 2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix} \quad (5.1\text{-}34)$$

$$\{M\} = [B] \left\{ \quad '' \quad \right\} - [D] \left\{ \quad '' \quad \right\} \quad (5.1\text{-}35)$$

با قرار دادن این روابط در معادلهٔ حاکم (5.1-21) داریم:

$$
\begin{aligned}
&A_{11}\left(\frac{\partial^2 u_0}{\partial x^2}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2}\right)+A_{12}\left(\frac{\partial^2 v_0}{\partial x\partial y}+\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial x\partial y}\right)\\
&+A_{16}\left(\frac{\partial^2 u_0}{\partial x\partial y}+\frac{\partial^2 v_0}{\partial x^2}+\frac{\partial^2 w_0}{\partial x^2}\frac{\partial w_0}{\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x\partial y}\right)\\
&-B_{11}\frac{\partial^3 w_0}{\partial x^3}-B_{12}\frac{\partial^3 w_0}{\partial x\partial y^2}-2B_{16}\frac{\partial^3 w_0}{\partial x^2\partial y}\\
&+A_{16}\left(\frac{\partial^2 u_0}{\partial x\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x\partial y}\right)+A_{26}\left(\frac{\partial^2 v_0}{\partial y^2}+\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y^2}\right)\\
&+A_{66}\left(\frac{\partial^2 u_0}{\partial y^2}+\frac{\partial^2 v_0}{\partial x\partial y}+\frac{\partial^2 w_0}{\partial x\partial y}\frac{\partial w_0}{\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2}\right)\\
&-B_{16}\frac{\partial^3 w_0}{\partial x^2\partial y}-B_{26}\frac{\partial^3 w_0}{\partial y^3}-2B_{66}\frac{\partial^3 w_0}{\partial x\partial y^2}\\
&-\left(\frac{\partial N_{xx}^T}{\partial x}+\frac{\partial N_{xy}^T}{\partial y}\right)=I_0\frac{\partial^2 u_0}{\partial t^2}-I_1\frac{\partial^3 w_0}{\partial x\partial t^2}
\end{aligned}
$$

(5.1-36a)

$$
\begin{aligned}
&A_{16}\left(\frac{\partial^2 u_0}{\partial x^2}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2}\right)+A_{26}\left(\frac{\partial^2 v_0}{\partial x\partial y}+\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial x\partial y}\right)\\
&+A_{66}\left(\frac{\partial^2 u_0}{\partial x\partial y}+\frac{\partial^2 v_0}{\partial x^2}+\frac{\partial^2 w_0}{\partial x^2}\frac{\partial w_0}{\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x\partial y}\right)\\
&-B_{16}\frac{\partial^3 w_0}{\partial x^3}-B_{26}\frac{\partial^3 w_0}{\partial x\partial y^2}-2B_{66}\frac{\partial^3 w_0}{\partial x^2\partial y}\\
&+A_{12}\left(\frac{\partial^2 u_0}{\partial x\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x\partial y}\right)+A_{22}\left(\frac{\partial^2 v_0}{\partial y^2}+\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y^2}\right)\\
&+A_{26}\left(\frac{\partial^2 u_0}{\partial y^2}+\frac{\partial^2 v_0}{\partial x\partial y}+\frac{\partial^2 w_0}{\partial x\partial y}\frac{\partial w_0}{\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2}\right)\\
&-B_{12}\frac{\partial^3 w_0}{\partial x^2\partial y}-B_{22}\frac{\partial^3 w_0}{\partial y^3}-2B_{26}\frac{\partial^3 w_0}{\partial x\partial y^2}\\
&-\left(\frac{\partial N_{xy}^T}{\partial x}+\frac{\partial N_{yy}^T}{\partial y}\right)=I_0\frac{\partial^2 v_0}{\partial t^2}-I_1\frac{\partial^3 w_0}{\partial y\partial t^2}
\end{aligned}
\qquad (5.1\text{-}36b)
$$

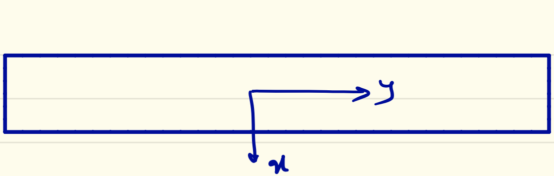
$$
\begin{aligned}
&B_{11}\left(\frac{\partial^3 u_0}{\partial x^3}+\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial x^2}+\frac{\partial w_0}{\partial x}\frac{\partial^3 w_0}{\partial x^3}\right)+B_{12}\left(\frac{\partial^3 v_0}{\partial x^2\partial y}+\frac{\partial^2 w_0}{\partial x\partial y}\frac{\partial^2 w_0}{\partial x\partial y}\right.\\
&\left.+\frac{\partial w_0}{\partial y}\frac{\partial^3 w_0}{\partial x^2\partial y}\right)+B_{16}\left(\frac{\partial^3 u_0}{\partial x^2\partial y}+\frac{\partial^3 v_0}{\partial x^3}+\frac{\partial^3 w_0}{\partial x^3}\frac{\partial w_0}{\partial y}+2\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial x\partial y}\right.\\
&\left.+\frac{\partial w_0}{\partial x}\frac{\partial^3 w_0}{\partial x^2\partial y}\right)-D_{11}\frac{\partial^4 w_0}{\partial x^4}-D_{12}\frac{\partial^4 w_0}{\partial x^2\partial y^2}-2D_{16}\frac{\partial^4 w_0}{\partial x^3\partial y}\\
&+2B_{16}\left(\frac{\partial^3 u_0}{\partial x^2\partial y}+\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial x\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^3 w_0}{\partial x^2\partial y}\right)+2B_{26}\left(\frac{\partial^3 v_0}{\partial x\partial y^2}+\frac{\partial^2 w_0}{\partial x\partial y}\frac{\partial^2 w_0}{\partial y^2}\right.\\
&\left.+\frac{\partial w_0}{\partial y}\frac{\partial^3 w_0}{\partial x\partial y^2}\right)+2B_{66}\left(\frac{\partial^3 u_0}{\partial x\partial y^2}+\frac{\partial^3 v_0}{\partial x^2\partial y}+\frac{\partial^3 w_0}{\partial x^2\partial y}\frac{\partial w_0}{\partial y}+\frac{\partial^2 w_0}{\partial x\partial y}\frac{\partial^2 w_0}{\partial x\partial y}\right.\\
&\left.+\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2}+\frac{\partial w_0}{\partial x}\frac{\partial^3 w_0}{\partial x\partial y^2}\right)-2D_{16}\frac{\partial^4 w_0}{\partial x^3\partial y}-2D_{26}\frac{\partial^4 w_0}{\partial x\partial y^3}\\
&-4D_{66}\frac{\partial^4 w_0}{\partial x^2\partial y^2}+B_{12}\left(\frac{\partial^3 u_0}{\partial x\partial y^2}+\frac{\partial^2 w_0}{\partial x\partial y}\frac{\partial^2 w_0}{\partial x\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^3 w_0}{\partial x\partial y^2}\right)
\end{aligned}
$$

(5.1.36c)

$$
\begin{aligned}
&+B_{22}\left(\frac{\partial^3 v_0}{\partial y^3}+\frac{\partial^2 w_0}{\partial y^2}\frac{\partial^2 w_0}{\partial y^2}+\frac{\partial w_0}{\partial y}\frac{\partial^3 w_0}{\partial y^3}\right)+B_{26}\left(\frac{\partial^3 u_0}{\partial y^3}+\frac{\partial^3 v_0}{\partial x\partial y^2}\right.\\
&\left.+\frac{\partial^3 w_0}{\partial x\partial y^2}\frac{\partial w_0}{\partial y}+2\frac{\partial^2 w_0}{\partial x\partial y}\frac{\partial^2 w_0}{\partial y^2}+\frac{\partial w_0}{\partial x}\frac{\partial^3 w_0}{\partial y^3}\right)\\
&-D_{12}\frac{\partial^4 w_0}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4 w_0}{\partial y^4}-2D_{26}\frac{\partial^4 w_0}{\partial x\partial y^3}+\mathcal{N}(w_0)+q\\
&-\left(\frac{\partial^2 M_{xx}^T}{\partial x^2}+2\frac{\partial^2 M_{xy}^T}{\partial y\partial x}+\frac{\partial^2 M_{yy}^T}{\partial y^2}\right)\\
&=I_0\frac{\partial^2 w_0}{\partial t^2}-I_2\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w_0}{\partial x^2}+\frac{\partial^2 w_0}{\partial y^2}\right)+I_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y}\right)
\end{aligned}
$$

مثال: معادلات حاکم خمش استوانه‌ای:

cylindrical bending



$$\frac{\partial}{\partial y} = 0$$

با توجه به معادلات (5.1-36) داریم:

$$A_{11}\left(\frac{\partial^2 u_0}{\partial x^2} + \frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2}\right) + A_{16}\frac{\partial^2 v_0}{\partial x^2} - B_{11}\frac{\partial^3 w_0}{\partial x^3} - \frac{\partial N_{xx}^T}{\partial x} = I_0\frac{\partial^2 u_0}{\partial t^2} - I_1\frac{\partial^3 w_0}{\partial x \partial t^2}$$

$$A_{16}\left(\frac{\partial^2 u_0}{\partial x^2} + \frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2}\right) + A_{66}\frac{\partial^2 v_0}{\partial x^2} - B_{16}\frac{\partial^3 w_0}{\partial x^3} - \frac{\partial N_{xy}^T}{\partial x} = I_0\frac{\partial^2 v_0}{\partial t^2}$$

$$B_{11}\left(\frac{\partial^3 u_0}{\partial x^3} + \frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial w_0}{\partial x}\frac{\partial^3 w_0}{\partial x^3}\right) + B_{16}\frac{\partial^3 v_0}{\partial x^3} - D_{11}\frac{\partial^4 w_0}{\partial x^4} - \frac{\partial^2 M_{xx}^T}{\partial x^2}$$
$$+ \frac{\partial}{\partial x}\left(N_{xx}\frac{\partial w_0}{\partial x}\right) + q = I_0\frac{\partial^2 w_0}{\partial t^2} - I_2\frac{\partial^4 w_0}{\partial x^2 \partial t^2} + I_1\frac{\partial^3 u_0}{\partial x \partial t^2}$$

(5.1-37)

5-2- The First-Order Laminated Plate Theory (FSDT)

first-order shear deformation laminated plate theory (FSDT)

5-2-1- Displacements and strains

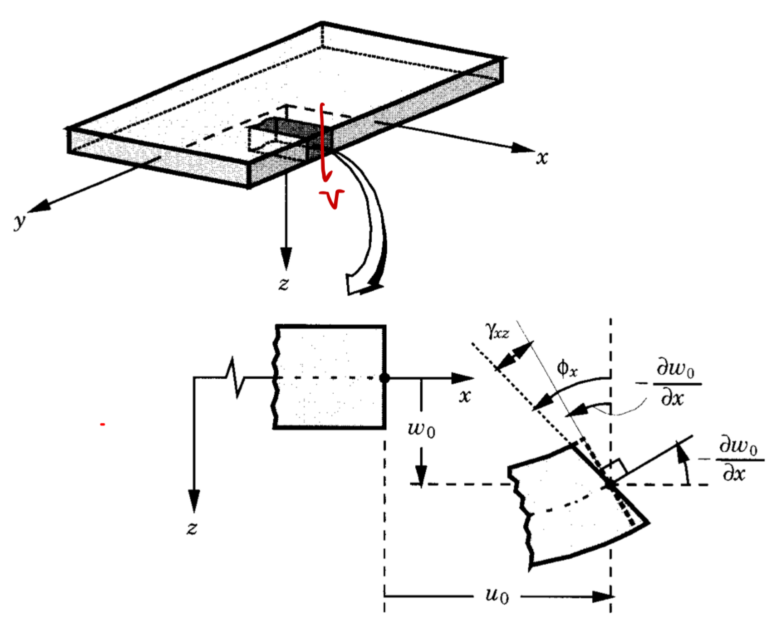
$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\,\phi_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) + z\,\phi_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \qquad (5.2\text{-}1)$$

تعداد مجهولات در این تئوری ۵ تا است. $(u_0, v_0, w_0, \phi_x, \phi_y)$

را جابجایی های عمومی (generalized displacement

تعریف

$$\phi_x = \frac{\partial u}{\partial z} \quad , \quad \phi_y = \frac{\partial v}{\partial z} \qquad (5.2\text{-}2)$$



در تئوری CLPT مقدار $\gamma_{xz}=0$ بود

اما در این تئوری (FSDT) داریم:

$$\gamma_{xz} = \phi_x - \left(-\frac{\partial w_0}{\partial x}\right) \neq 0 \quad (5.2\text{-}3)$$

ولی مقدار γ_{xz} در کل سطح مقطع ثابت است (تقریب)

$$\gamma_{xz} = \frac{\tau_{xz}}{G_{xz}} = \frac{V/A}{G_{xz}}$$

اگر $\frac{a}{h} \geq 50$ باشد (a: ابعاد صفحه، h: ضخامت)

$$\phi_x = -\frac{\partial w_0}{\partial x} \quad \text{و} \quad \phi_y = -\frac{\partial w_0}{\partial y}$$