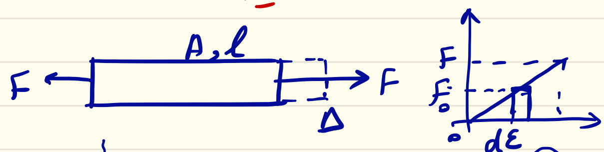
بسم الله الرحمن الرحیم

مواد مرکب

جلسه ۱۸

$\delta U$ انرژی کرنشی مجازی




$$\delta U = \int_V \{\sigma\}^T \cdot \{\delta \varepsilon\}\, dV$$

$$W = F.\Delta \rightsquigarrow W_0 = \frac{W}{V} = \frac{F.\Delta}{A.l}$$

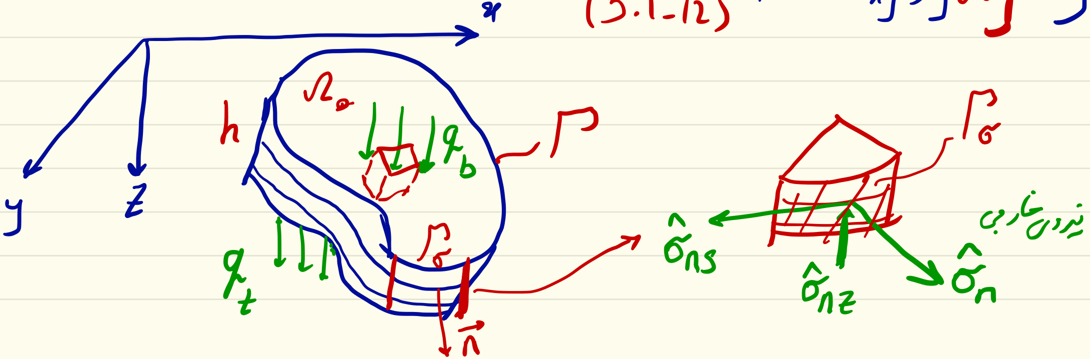
$$W_0 = \sigma . \varepsilon \rightsquigarrow dW_0 = \sigma . d\varepsilon$$

$$= \int_{\Omega_0}\int_{-h/2}^{h/2} (\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + 2\sigma_{xy}\delta\varepsilon_{xy})\, dz\,dx\,dy$$

$$\delta U_0 = \sigma\,\delta\varepsilon$$

$$\delta U = \int \delta U_0\, dV$$

$$= \int_{\Omega_0}\left\{\int_{-h/2}^{h/2} \left[\sigma_x(\delta\varepsilon_x^0 + z\,\delta\varepsilon_x^{(1)}) + \sigma_y(\delta\varepsilon_y^0 + z\,\delta\varepsilon_y^{(1)}) + \sigma_{xy}(\delta\gamma_{xy}^0 + z\,\delta\gamma_{xy}^{(1)})\right] dz\right\} dx\,dy \quad (5.1\text{-}12)$$



$\delta V$ انرژی پتانسیل نیروهای خارجی مجازی = منفی کار انجام شده توسط نیروهای خارجی

$$V = \vec{F}\cdot\vec{u} \sim \delta V = \vec{F}\cdot\delta\vec{u}$$

$$\delta V = -\int_{\Omega_0}\left[q_b(x,y)\,\delta w(x,y,h/2) + q_t(x,y)\,\delta w(x,y,-h/2)\right]dx\,dy$$

$$-\int_{\Gamma_\sigma}\int_{-h/2}^{h/2}\left[\hat{\sigma}_n\,\delta u_n + \hat{\sigma}_{ns}\,\delta u_s + \hat{\sigma}_{nz}\,\delta w\right]dz\,ds$$

$$= \int_{\Omega_0}\left[\left(q_b(x,y) + q_t(x,y)\right)\delta w_0(x,y)\right]dx\,dy$$

$$-\int_{\Gamma_\sigma}\int_{-h/2}^{h/2}\left[\hat{\sigma}_n\left(\delta u_{0n} - z\frac{\partial \delta w_0}{\partial n}\right) + \hat{\sigma}_{ns}\left(\delta u_{0s} - z\frac{\partial \delta w_0}{\partial s}\right) + \hat{\sigma}_{nz}\,\delta w_0\right]dz\,ds \qquad (5.1\text{-}13)$$

$\delta K$

انرژی جنبشی مجازی

$$K = \int_{V} \frac{1}{2}\rho(\dot{u}^2+\dot{v}^2+\dot{w}^2)\,dV \rightarrow \delta K = \int_{V} \rho(\dot{u}\,\delta\dot{u}+\dot{v}\,\delta\dot{v}+\dot{w}\,\delta\dot{w})\,dV$$

$$\delta K = \int_{\Omega_0}\int_{-h/2}^{h/2} \rho_0\Big[\Big(\dot{u}_0 - z\frac{\partial \dot{w}_0}{\partial x}\Big)\Big(\delta\dot{u}_0 - z\frac{\partial \delta\dot{w}_0}{\partial x}\Big)+\Big(\dot{v}_0 - z\frac{\partial \dot{w}_0}{\partial y}\Big)\Big(\delta\dot{v}_0 - z\frac{\partial \delta\dot{w}_0}{\partial y}\Big)$$

$$+\dot{w}_0\,\delta\dot{w}_0\Big]\,dz\,dx\,dy \qquad (5.1\text{-}14)$$

روابط (5.1-12, 13, 14) را در رابطه (5.1-11) قرار دهیم:

$$
\begin{aligned}
0 = \int_0^T \Bigg\{ \int_{\Omega_0} \Bigg[ & N_{xx}\delta\varepsilon_{xx}^{(0)} + M_{xx}\delta\varepsilon_{xx}^{(1)} + N_{yy}\delta\varepsilon_{yy}^{(0)} + M_{yy}\delta\varepsilon_{yy}^{(1)} + N_{xy}\delta\gamma_{xy}^{(0)} \\
& + M_{xy}\delta\gamma_{xy}^{(1)} - q\delta w_0 - I_0\left(\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + \dot{w}_0\delta\dot{w}_0\right) \\
& + I_1\left(\frac{\partial\delta\dot{w}_0}{\partial x}\dot{u}_0 + \frac{\partial\dot{w}_0}{\partial x}\delta\dot{u}_0 + \frac{\partial\delta\dot{w}_0}{\partial y}\dot{v}_0 + \frac{\partial\dot{w}_0}{\partial y}\delta\dot{v}_0\right) \\
& - I_2\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right)\Bigg] dxdy \\
& \underbrace{- \int_{\Gamma_\sigma}\left(\hat{N}_{nn}\delta u_{0n} + \hat{N}_{ns}\delta u_{0s} - \hat{M}_{nn}\frac{\partial\delta w_0}{\partial n} - \hat{M}_{ns}\frac{\partial\delta w_0}{\partial s} + \hat{Q}_n\delta w_0\right) ds}_{\text{عبارت مرزی}} \Bigg\} dt
\end{aligned}
\qquad (3.1\text{-}15)
$$

(3.3.19

در رابطه فوق داریم:

$$
\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} dz, \quad \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} z\, dz
$$

$$
\begin{Bmatrix} \hat{N}_{nn} \\ \hat{N}_{ns} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \hat{\sigma}_{nn} \\ \hat{\sigma}_{ns} \end{Bmatrix} dz, \quad \begin{Bmatrix} \hat{M}_{nn} \\ \hat{M}_{ns} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \hat{\sigma}_{nn} \\ \hat{\sigma}_{ns} \end{Bmatrix} z\, dz \qquad (3.1\text{-}16)
$$

$$
\begin{Bmatrix} I_0 \\ I_1 \\ I_2 \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} 1 \\ z \\ z^2 \end{Bmatrix} \rho_0\, dz, \qquad \hat{Q}_n = \int_{-\frac{h}{2}}^{\frac{h}{2}} \hat{\sigma}_{nz}\, dz
$$

حال با استفاده از رابطه (5.1-6) داریم:

$$\delta\varepsilon_x^0 = \frac{\partial \delta u_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial \delta w_0}{\partial x} \quad , \quad \delta\varepsilon_x^{(1)} = -\frac{\partial^2 \delta w_0}{\partial x^2}$$

$$\delta\varepsilon_y^0 = \frac{\partial \delta v_0}{\partial y} + \frac{\partial w_0}{\partial y}\frac{\partial \delta w_0}{\partial y} \quad , \quad \delta\varepsilon_y^{(1)} = -\frac{\partial^2 \delta w_0}{\partial y^2}$$

$$\delta\gamma_{xy}^0 = \frac{\partial \delta u_0}{\partial y} + \frac{\partial \delta v_0}{\partial x} + \frac{\partial \delta w_0}{\partial x}\frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial x}\frac{\partial \delta w_0}{\partial y} \quad , \quad \delta\gamma_{xy}^{(1)} = -2\frac{\partial^2 \delta w_0}{\partial x \partial y}$$

با جایگزاری این رابطه (5.1-17) در رابطه (5.1-15) داریم:

$$
\begin{aligned}
0 = \int_0^T \Bigg\{ \int_{\Omega_0} \Bigg[ & - N_{xx,x}\delta u_0 - \left(N_{xx}\frac{\partial w_0}{\partial x}\right)_{,x}\delta w_0 - M_{xx,xx}\delta w_0 - N_{yy,y}\delta v_0 \\
& - \left(N_{yy}\frac{\partial w_0}{\partial y}\right)_{,y}\delta w_0 - M_{yy,yy}\delta w_0 - N_{xy,y}\delta u_0 - N_{xy,x}\delta v_0 \\
& - \left(N_{xy}\frac{\partial w_0}{\partial y}\right)_{,x}\delta w_0 - \left(N_{xy}\frac{\partial w_0}{\partial x}\right)_{,y}\delta w_0 - 2M_{xy,xy}\delta w_0 - q\delta w_0 \\
& + I_0\left(\ddot{u}_0\delta u_0 + \ddot{v}_0\delta v_0 + \ddot{w}_0\delta w_0\right) - I_2\left(\frac{\partial^2 \ddot{w}_0}{\partial x^2} + \frac{\partial^2 \ddot{w}_0}{\partial y^2}\right)\delta w_0 \\
& + I_1\left(\frac{\partial \ddot{u}_0}{\partial x}\delta w_0 - \frac{\partial \ddot{w}_0}{\partial x}\delta u_0 + \frac{\partial \ddot{v}_0}{\partial y}\delta w_0 - \frac{\partial \ddot{w}_0}{\partial y}\delta v_0\right)\Bigg] dxdy \\
+ \oint_\Gamma \Bigg[ & N_{xx}n_x\delta u_0 + \left(N_{xx}\frac{\partial w_0}{\partial x}\right)n_x\delta w_0 - M_{xx}n_x\frac{\partial \delta w_0}{\partial x} + M_{xx,x}n_x\delta w_0 \\
& + N_{yy}n_y\delta v_0 + \left(N_{yy}\frac{\partial w_0}{\partial y}\right)n_y\delta w_0 - M_{yy}n_y\frac{\partial \delta w_0}{\partial y} + M_{yy,y}n_y\delta w_0 \\
& - M_{xy}n_x\frac{\partial \delta w_0}{\partial y} + M_{xy,x}n_y\delta w_0 - M_{xy}n_y\frac{\partial \delta w_0}{\partial x} + M_{xy,y}n_x\delta w_0 \\
& + N_{xy}n_y\delta u_0 + N_{xy}n_x\delta v_0 + N_{xy}\frac{\partial w_0}{\partial y}n_x\delta w_0 + N_{xy}\frac{\partial w_0}{\partial x}n_y\delta w_0\Bigg] ds \\
- \int_{\Gamma_\sigma} & \left(\hat{N}_{nn}\delta u_{0n} + \hat{N}_{ns}\delta u_{0s} - \hat{M}_{nn}\frac{\partial \delta w_0}{\partial n} - \hat{M}_{ns}\frac{\partial \delta w_0}{\partial s} + \hat{Q}_n\delta w_0\right) ds \\
+ \oint_\Gamma & \left[-I_1\left(\ddot{u}_0 n_x + \ddot{v}_0 n_y\right) + I_2\left(\frac{\partial \ddot{w}_0}{\partial x}n_x + \frac{\partial \ddot{w}_0}{\partial y}n_y\right)\right]\delta w_0 \, ds \Bigg\} dt \quad (3.3.22)
\end{aligned}
$$

(5.1-18)

با کمی مرتب سازی داریم:

$$
\begin{aligned}
0 = \int_0^T \Bigg\{ \int_{\Omega_0} \Bigg[ & - \left( N_{xx,x} + N_{xy,y} - I_0 \ddot{u}_0 + I_1 \frac{\partial \ddot{w}_0}{\partial x} \right) \delta u_0 \\
& - \left( N_{xy,x} + N_{yy,y} - I_0 \ddot{v}_0 + I_1 \frac{\partial \ddot{w}_0}{\partial y} \right) \delta v_0 \\
& - \Bigg( M_{xx,xx} + 2M_{xy,xy} + M_{yy,yy} + \mathcal{N}(w_0) + q \\
& - I_0 \ddot{w}_0 - I_1 \frac{\partial \ddot{u}_0}{\partial x} - I_1 \frac{\partial \ddot{v}_0}{\partial y} + I_2 \frac{\partial^2 \ddot{w}_0}{\partial x^2} + I_2 \frac{\partial^2 \ddot{w}_0}{\partial y^2} \Bigg) \delta w_0 \Bigg] dxdy \\
+ \int_{\Gamma_\sigma} \Bigg[ & \left( N_{xx} n_x + N_{xy} n_y \right) \delta u_0 + \left( N_{xy} n_x + N_{yy} n_y \right) \delta v_0 \\
& + \Bigg( M_{xx,x} n_x + M_{xy,y} n_x + M_{yy,y} n_y + M_{xy,x} n_y + \mathcal{P}(w_0) \\
& - I_1 \ddot{u}_0 n_x - I_1 \ddot{v}_0 n_y + I_2 \frac{\partial \ddot{w}_0}{\partial x} n_x + I_2 \frac{\partial \ddot{w}_0}{\partial y} n_y \Bigg) \delta w_0 \\
& - \left( M_{xx} n_x + M_{xy} n_y \right) \frac{\partial \delta w_0}{\partial x} - \left( M_{xy} n_x + M_{yy} n_y \right) \frac{\partial \delta w_0}{\partial y} \Bigg] ds \\
- \int_{\Gamma_\sigma} & \left( \hat{N}_{nn} \delta u_{0n} + \hat{N}_{ns} \delta u_{0s} - \hat{M}_{nn} \frac{\partial \delta w_0}{\partial n} - \hat{M}_{ns} \frac{\partial \delta w_0}{\partial s} + \hat{Q}_n \delta w_0 \right) ds \Bigg\} dt
\end{aligned}
\qquad (5.1\text{-}19)
$$

$$
\begin{aligned}
\mathcal{N}(w_0) &= \frac{\partial}{\partial x} \left( N_{xx} \frac{\partial w_0}{\partial x} + N_{xy} \frac{\partial w_0}{\partial y} \right) + \frac{\partial}{\partial y} \left( N_{xy} \frac{\partial w_0}{\partial x} + N_{yy} \frac{\partial w_0}{\partial y} \right) \\
\mathcal{P}(w_0) &= \left( N_{xx} \frac{\partial w_0}{\partial x} + N_{xy} \frac{\partial w_0}{\partial y} \right) n_x + \left( N_{xy} \frac{\partial w_0}{\partial x} + N_{yy} \frac{\partial w_0}{\partial y} \right) n_y
\end{aligned}
\qquad (5.1\text{-}20)
$$

اگر حالت خطی در نظر گرفته شود مقادیر $\mathcal{N}$ و $\mathcal{P}$ صفر می‌شوند.

پس داریم:

(5.1-21)

معادلات حاکم بر ورق CLPT

$$\delta u_0: \quad \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \frac{\partial^2 u_0}{\partial t^2} - I_1 \frac{\partial^2}{\partial t^2}\left(\frac{\partial w_0}{\partial x}\right)$$

$$\delta v_0: \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \frac{\partial^2 v_0}{\partial t^2} - I_1 \frac{\partial^2}{\partial t^2}\left(\frac{\partial w_0}{\partial y}\right)$$

$$\delta w_0: \quad \frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \mathcal{N}(w_0) + q = I_0 \frac{\partial^2 w_0}{\partial t^2} - I_2 \frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2}\right) + I_1 \frac{\partial^2}{\partial t^2}\left(\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y}\right)$$