بسم الله الرحمن الرحیم		مواد مرکب		جلسه ۱۷

Chapter 5: Classical and First_order Theories

فرض عمومی:

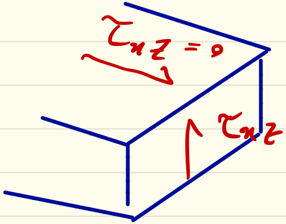
۱ـ تمام لایه‌ها بصورت کامل بهم متصل هستند.

۲ـ مواد تشکیل دهنده لایه‌ها الاستیک خطی هستند و همچنین ارتوتروپ می باشند.

۳ـ ضخامت لایه‌ها ثابت است

۴ـ تغییر شکل های کوچک

۵ـ تنش برشی در صفحه مرزی بالا و پایین صفر است



5.1. The classical Laminated Plate Theory (CLPT)

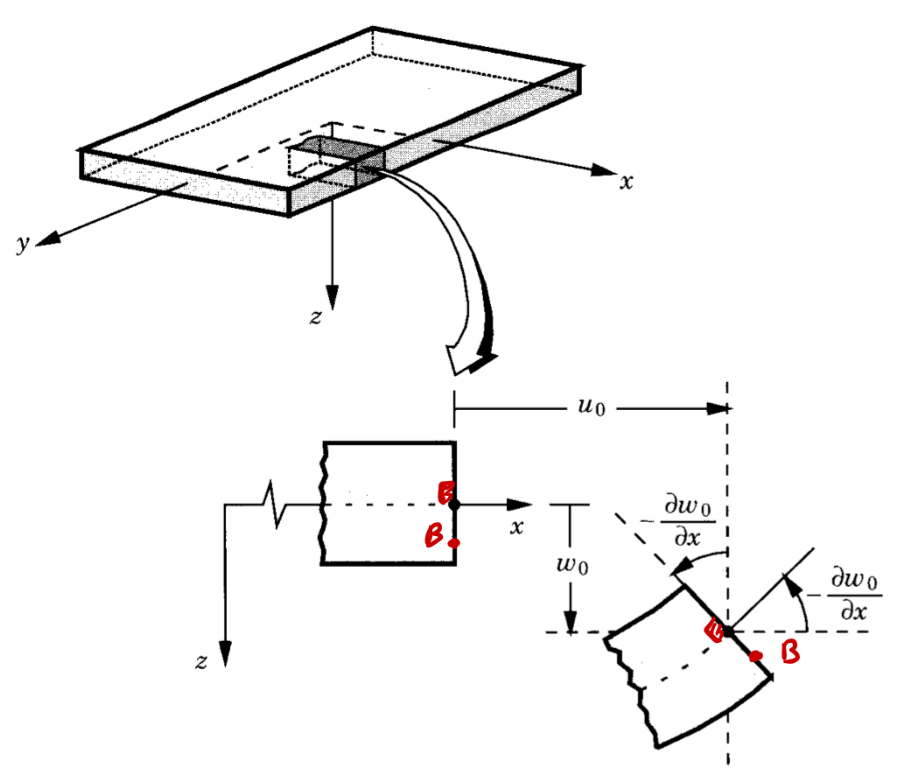
5.1.1. Displacements and Strains

$$
\begin{aligned}
u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} \\
v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial y} \qquad (5.1\text{-}1) \\
w(x,y,z,t) &= w_0(x,y,t)
\end{aligned}
$$

فرضیات :

۱- سطوح مستوی عمود بر سطح میانی بعد از تغییر شکل مستوی و عمود بر سطح میانی باقی می مانند.

۲- تغییر شکل ها در جهت ضخامت قابل صرف نظر کردن است $(\varepsilon_z \simeq 0)$



x
y
z
u_0
B
B
x
$\frac{\partial w_0}{\partial x}$
w_0
z
$\frac{\partial w_0}{\partial x}$
B
B

کرنش های غیرخطی :

$$
\begin{aligned}
E_{xx} &= \frac{\partial u}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right] \\
E_{yy} &= \frac{\partial v}{\partial y} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] \\
E_{zz} &= \frac{\partial w}{\partial z} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] \\
E_{xy} &= \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right) \\
E_{xz} &= \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} + \frac{\partial u}{\partial x}\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial z} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial z}\right) \\
E_{yz} &= \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} + \frac{\partial u}{\partial y}\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y}\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\frac{\partial w}{\partial z}\right)
\end{aligned}
\qquad (5.1\text{-}2)
$$

$$
\frac{\partial u}{\partial x},\ \frac{\partial u}{\partial y},\ \frac{\partial v}{\partial x},\ \frac{\partial v}{\partial y},\ \frac{\partial w}{\partial z} = O(\epsilon) \qquad (5.1\text{-}3)
$$

اما مقادیر زیر درجه 2 خطا هستند (قابل صرفنظر کردن)

trains. Terms of order ϵ^2 are

$$\left(\frac{\partial u}{\partial x}\right)^2, \left(\frac{\partial u}{\partial y}\right)^2, \left(\frac{\partial u}{\partial z}\right)^2, \left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right), \left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial u}{\partial z}\right), \left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial u}{\partial z}\right)$$

$$\left(\frac{\partial v}{\partial x}\right)^2, \left(\frac{\partial v}{\partial y}\right)^2, \left(\frac{\partial v}{\partial z}\right)^2, \left(\frac{\partial v}{\partial x}\right)\left(\frac{\partial v}{\partial y}\right), \left(\frac{\partial v}{\partial x}\right)\left(\frac{\partial v}{\partial z}\right), \left(\frac{\partial v}{\partial y}\right)\left(\frac{\partial v}{\partial z}\right)$$

$$\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial z}\right), \left(\frac{\partial w}{\partial y}\right)\left(\frac{\partial w}{\partial z}\right), \left(\frac{\partial w}{\partial z}\right)^2$$

(5.1-4a)

مقادیر زیر با اینکه کوچک هستند اما قابل صرفنظر کردن نیستند

$$\left(\frac{\partial w}{\partial x}\right)^2 \text{ و } \left(\frac{\partial w}{\partial y}\right)^2 \text{ و } \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}$$

درنتیجه برای کرنش ها و چرخش های کوچک می توان گفت: (کرنش های غیرخطی)

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \text{ و } \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \text{ و } \varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2$$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \text{ و } \varepsilon_{zz} = \frac{\partial w}{\partial z} \quad (5.1\text{-}5)$$

با قرار دادن رابطه (5.1-5) در (5.1-1) داریم:

$$
\begin{aligned}
\varepsilon_{xx} &= \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 - z\frac{\partial^2 w_0}{\partial x^2} \\
\varepsilon_{xy} &= \frac{1}{2}\left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right) - z\frac{\partial^2 w_0}{\partial x \partial y} \\
\varepsilon_{yy} &= \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - z\frac{\partial^2 w_0}{\partial y^2} \\
\varepsilon_{xz} &= \frac{1}{2}\left(-\frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial x}\right) = 0 \\
\varepsilon_{yz} &= \frac{1}{2}\left(-\frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial y}\right) = 0 \\
\varepsilon_{zz} &= 0
\end{aligned}
\tag{5.1-6}
$$

کرنش های von Karman

و ورقی که با این فرضیات حل شود را ورق

فون کارمن می نامند.

$$
\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \varepsilon_x^{(1)} \\ \varepsilon_y^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} \tag{5.1-7}
$$

$$
\{\varepsilon^0\} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \end{Bmatrix} \quad \text{و} \quad \{\varepsilon^{(1)}\} = \begin{Bmatrix} -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix} \tag{5.1-8}
$$

$\{\varepsilon^{\circ}\}$: membrane Strain

$\{\varepsilon^{(1)}\}$: bending strain , curvature

5.1-2 ـ روابط فضای حالت چندلایه (CLPT)

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix}_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_1 - \alpha_1 \Delta T \\ \varepsilon_2 - \alpha_2 \Delta T \\ \varepsilon_6 \end{Bmatrix}_k - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & 0 \end{bmatrix}_k \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}_k \quad (5.1\text{-}9)$$

ماتریس $[e]_k$ را مدول پیزوالکتریک می‌نامند و بردار $\{E\}$ را
میدان الکتریکی می‌نامند.

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}^{(k)} = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}^{(k)} \left(\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} - \begin{Bmatrix}\alpha_{xx}\\ \alpha_{yy}\\ 2\alpha_{xy}\end{Bmatrix}\Delta T\right) - \begin{bmatrix}0 & 0 & \bar{e}_{31}\\ 0 & 0 & \bar{e}_{32}\\ 0 & 0 & \bar{e}_{36}\end{bmatrix}^{(k)} \begin{Bmatrix}\mathcal{E}_x\\ \mathcal{E}_y\\ \mathcal{E}_z\end{Bmatrix}^{(k)}$$

برای جسب های غیراصلی

(5.1-10)

5.1.3 _ معادلات حرکت.

$$\int_0^T (\delta U + \delta \bar{V} - \delta K)\, dt = 0 \qquad (5.1\text{-}11)$$

δU : انرژی کرنشی مجازی کل جسم

$\delta \bar{V}$: کار مجازی انجام شده توسط نیروهای خارجی

δK : انرژی جنبشی مجازی

$$I = \int L\, dt = \int (\Pi - K)\, dt = \text{مینیمم}$$

$$\Rightarrow \delta I = 0 \longrightarrow \int (\delta \Pi - \delta K)\, dt = 0$$