بسم الله الرحمن الرحیم   مواد مرکب   جلسه ۱۳

فصل چهارم - ورق‌ها

فرض‌های کیرشهف :

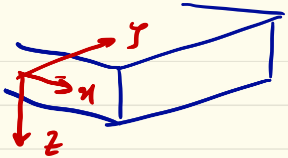
۱- خیز سطح میانی، w، در مقایسه با ضخامت کوچک است. همچنین شیب ورق نیز کوچک می‌باشد.

۲- در اثر خمش سطح میانی ورق بدون کرنش باقی می‌ماند

۳- صفحات عمود بر سطح میانی، بعد از خمش عمود بر این سطح و مستوی باقی می‌مانند. این فرض به معنی این است که γ_{xz} و γ_{yz} قابل صرفنظر کردن هستند.

۴- تنش عمود بر سطح میانی (σ_z) صفر می‌باشد. همچنین ضخامت ورق تقریبا ثابت باقی می‌ماند. یعنی ε_z قابل صرفنظر کردن می‌باشد.

خمش ورق‌ها:



$$\begin{cases} w = w(x,y) \\ u = u_0(x,y) - z\dfrac{\partial w}{\partial x} \\ v = v_0(x,y) - z\dfrac{\partial w}{\partial y} \end{cases} \Rightarrow \begin{cases} w = w(x,y) \\ u = -z\dfrac{\partial w}{\partial x} \\ v = -z\dfrac{\partial w}{\partial y} \end{cases} \Rightarrow \begin{cases} \varepsilon_x = -z\dfrac{\partial^2 w}{\partial x^2} \\ \varepsilon_y = -z\dfrac{\partial^2 w}{\partial y^2} \\ \gamma_{xy} = -2z\dfrac{\partial^2 w}{\partial x \partial y} \end{cases}$$

یا

$$\begin{cases} \varepsilon_x = z k_x \\ \varepsilon_y = z k_y \\ \gamma_{xy} = 2 z k_{xy} \end{cases}$$

1-4- یافتن تنش‌ها براساس جابجایی‌ها در مواد ایزوتروپ

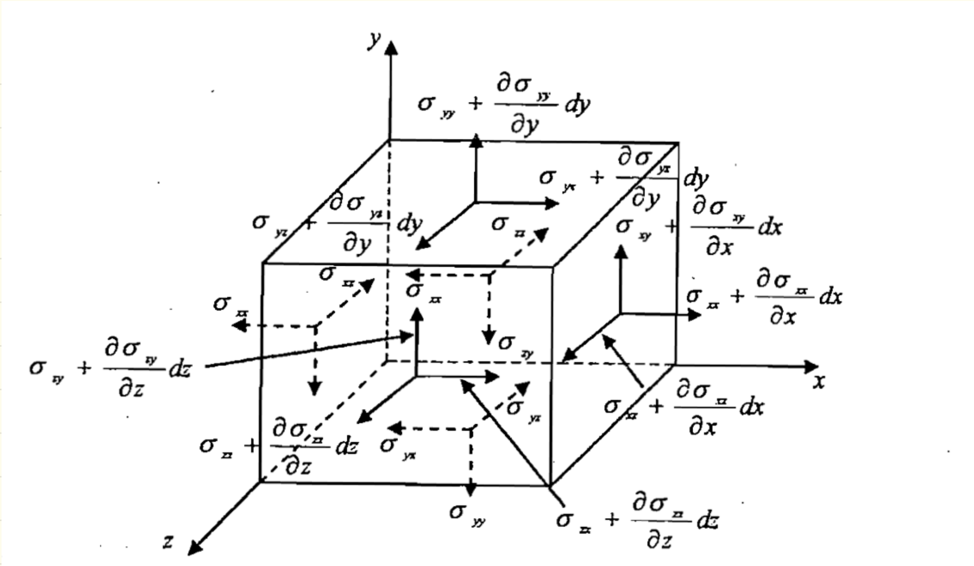
روابط هوک

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

$$\tau_{xy} = G\gamma_{xy}$$

$$\Rightarrow \begin{cases} \sigma_x = \dfrac{Ez}{1-\nu^2}(k_x + \nu k_y) = -\dfrac{Ez}{1-\nu^2}\left(\dfrac{\partial^2 w}{\partial x^2} + \nu\dfrac{\partial^2 w}{\partial y^2}\right) \\ \sigma_y = \dfrac{Ez}{1-\nu^2}(k_y + \nu k_x) = -\dfrac{Ez}{1-\nu^2}\left(\dfrac{\partial^2 w}{\partial y^2} + \nu\dfrac{\partial^2 w}{\partial x^2}\right) \\ \tau_{xy} = \dfrac{Ez}{1+\nu}k_{xy} = -\dfrac{Ez}{1+\nu}\dfrac{\partial^2 w}{\partial x \partial y} \end{cases} \quad (I)$$



روابط تعادل کلی :

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{yx}}{\partial y}+\frac{\partial\sigma_{zx}}{\partial z}+F_x=0$$
$$\frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{zy}}{\partial z}+F_y=0$$
$$\frac{\partial\sigma_{xz}}{\partial x}+\frac{\partial\sigma_{yz}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}+F_z=0$$

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}+\frac{\partial\sigma_{xz}}{\partial z}=0$$
$$\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yz}}{\partial z}=0$$
$$\frac{\partial\sigma_{zz}}{\partial z}+\frac{\partial\sigma_{xz}}{\partial x}+\frac{\partial\sigma_{yz}}{\partial y}=0$$

$\Rightarrow$

$$\sigma_{xz}=\int_z^{h/2}\left(\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}\right)dz=-\frac{E}{2(1-\nu^2)}\left(\frac{h^2}{4}-z^2\right)\left[\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)\right]$$
$$\sigma_{yz}=\int_z^{h/2}\left(\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{xy}}{\partial x}\right)dz=-\frac{E}{2(1-\nu^2)}\left(\frac{h^2}{4}-z^2\right)\left[\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)\right]$$

II
ایزوتروپی

$$\sigma_{zz}=-\frac{E}{2(1-\nu^2)}\left(\frac{h^3}{12}-\frac{h^3 z}{4}+\frac{z^3}{3}\right)\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)\right]$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} z\,dz$$

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \end{Bmatrix} dz$$

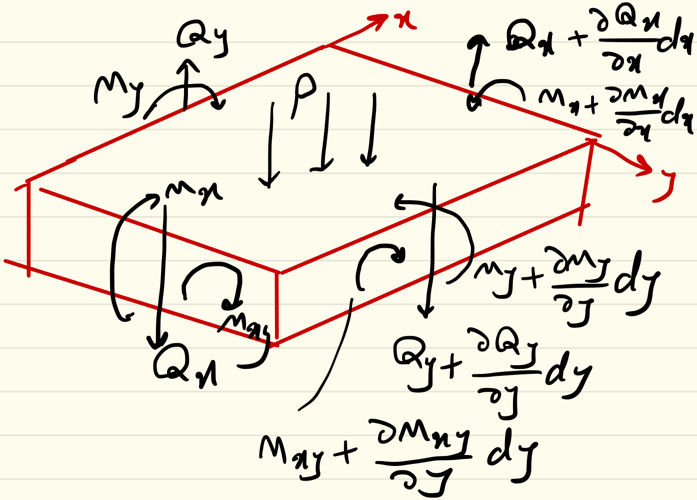
$$\Rightarrow \begin{cases} M_x = D(K_x + \nu K_y) = -D\left(\dfrac{\partial^2 w}{\partial x^2} + \nu \dfrac{\partial^2 w}{\partial y^2}\right) \\ M_y = D(K_y + \nu K_x) = -D\left(\dfrac{\partial^2 w}{\partial y^2} + \nu \dfrac{\partial^2 w}{\partial x^2}\right) \\ M_{xy} = D(1-\nu) K_{xy} = -D(1-\nu) \dfrac{\partial^2 w}{\partial x \partial y} \end{cases} \quad (\mathrm{III})$$

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

سفتی خمشی ورق

$$\begin{cases} \sigma_x = \dfrac{12 M_x z}{h^3} \\ \sigma_y = \dfrac{12 M_y z}{h^3} \\ \sigma_{xy} = \dfrac{12 M_{xy} z}{h^3} \end{cases}$$

۴-۲ - روابط تعادل در ورق‌های ایزوتروپ



$\Sigma F_z = 0$

$$\boxed{\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + P = 0} \quad (1)$$

$\Sigma M_{..} = 0$

$$\boxed{\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0} \quad (2)$$

$$\boxed{\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = 0} \quad (3)$$

$$\xrightarrow{2,3}\begin{cases} Q_x = -D\dfrac{\partial}{\partial x}\left(\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}\right) \\[2ex] Q_y = -D\dfrac{\partial}{\partial y}\left(\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}\right)\end{cases} \qquad \text{IV}$$

اگر از روابط (2) و (3) مقادیر Q_x و Q_y را بیابیم و در رابطه (1) قرار دهیم داریم:

$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\,\partial y}+\frac{\partial^2 M_y}{\partial y^2}=-P \xRightarrow{\text{III}} \frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{P}{D}$$

$$\boxed{\nabla^4 w=\frac{P}{D}}$$

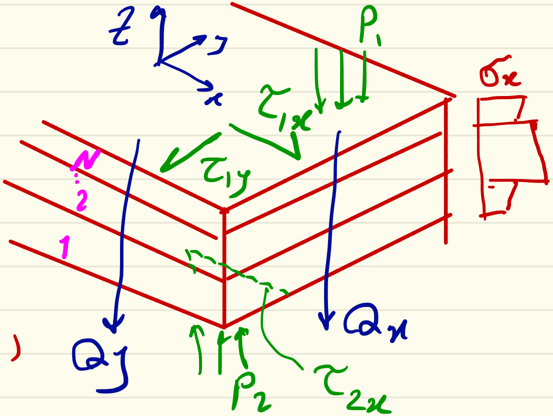
رابطه تعادل ورق ایزوتروپ

$$\nabla^4=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)=\nabla^2\nabla^2$$

3-4 ـ روابط تعادل ورق‌های چندلایه

$$\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \sigma_{yx}}{\partial y}+\frac{\partial \sigma_{zx}}{\partial z}+F_x=0$$
$$\frac{\partial \sigma_{xy}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \sigma_{zy}}{\partial z}+F_y=0 \qquad (a)$$
$$\frac{\partial \sigma_{xz}}{\partial x}+\frac{\partial \sigma_{yz}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}+F_z=0$$

$$\left|\begin{array}{l} \sigma_{xz}(h_N)=\tau_{1x} \\ \sigma_{xz}(h_0)=\tau_{2x} \\ \sigma_{zz}(h_N)=P_1 \\ \sigma_{zz}(h_0)=P_2 \end{array}\right.$$



$$\int (a)\,dz \Rightarrow \boxed{\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}+\tau_{1x}-\tau_{2x}=0} \qquad (4)$$

$$\boxed{\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}+\tau_{1y}-\tau_{2y}=0} \qquad (5)$$

$$\boxed{\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+P_1-P_2=0} \qquad (6)$$

$$\boxed{\frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}-Q_x+\frac{h}{2}\left(\tau_{1x}+\tau_{2x}\right)=0} \qquad (7)$$

$$\boxed{\frac{\partial M_y}{\partial y}+\frac{\partial M_{xy}}{\partial x}-Q_y+\frac{h}{2}\left(\tau_{1y}+\tau_{2y}\right)=0} \qquad (8)$$

اگر فقط خمش ورق را در نظر بگیریم:

$$\begin{cases} \dfrac{\partial M_x}{\partial x} + \dfrac{\partial M_{xy}}{\partial y} - Q_x = 0 \\[2ex] \dfrac{\partial M_{xy}}{\partial x} + \dfrac{\partial M_y}{\partial y} - Q_y = 0 \\[2ex] \dfrac{\partial Q_x}{\partial x} + \dfrac{\partial Q_y}{\partial y} + P(x,y) = 0 \end{cases} \Rightarrow \boxed{\dfrac{\partial^2 M_x}{\partial x^2} + \dfrac{\partial^2 M_{xy}}{\partial x \partial y} + \dfrac{\partial^2 M_y}{\partial y^2} + P(x,y) = 0} \quad (9)$$

معادله ورق عمومی (ایزوتروپ یا غیرایزوتروپ)

حالت خاص:

- $B_{ij} = 0$
- $(\ )_{16} = (\ )_{26} = 0$
- فقط خمش در اثر بار عرضی بدون تغییر دما و رطوبت

$$\begin{Bmatrix} N \\ \hdashline M \end{Bmatrix} = \left[\begin{array}{c:c} A & B \\ \hdashline B & D \end{array}\right] \begin{Bmatrix} \varepsilon^0 \\ \hdashline K \end{Bmatrix}$$

$$\Rightarrow \begin{cases} M_x = D_{11} K_x + D_{12} K_y \\ M_y = D_{12} K_x + D_{22} K_y \\ M_{xy} = 2 D_{66} K_{xy} \end{cases} \quad \text{یا} \quad \begin{cases} M_x = -D_{11} \dfrac{\partial^2 w}{\partial x^2} - D_{12} \dfrac{\partial^2 w}{\partial y^2} \\[2ex] M_y = -D_{12} \dfrac{\partial^2 w}{\partial x^2} - D_{22} \dfrac{\partial^2 w}{\partial y^2} \\[2ex] M_{xy} = -2 D_{66} \dfrac{\partial^2 w}{\partial x \partial y} \end{cases}$$

$$\xrightarrow{(9)} \quad D_{11}\cdot\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial^2 x\,\partial^2 y}+D_{22}\frac{\partial^4 w}{\partial y^4}=P(x,y)$$

$$\boxed{D_1\,\frac{\partial^4 w}{\partial x^4}+2D_3\frac{\partial^4 w}{\partial x^2\partial y^2}+D_2\frac{\partial^4 w}{\partial y^4}=P(x,y)} \quad (15)$$

معادله تعادل

ورق $B_{ij}=0$

$(\dots)_{16}=(\dots)_{26}=0$

$$D_{11}=D_1\ ,\quad D_{22}=D_2\ ,\quad (D_{12}+2D_{66})=D_3$$