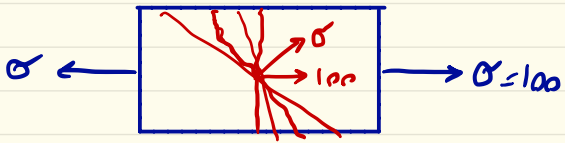
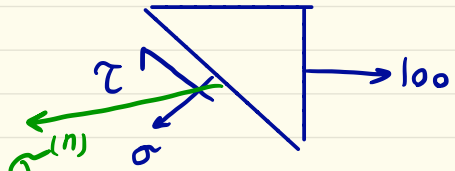
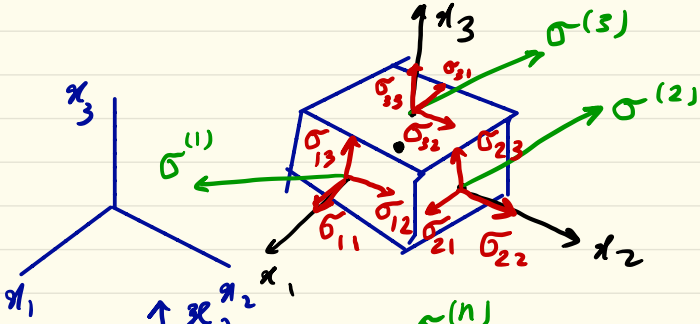
بسم الله الرحمن الرحیم صوا در کلب جلسه ۶

فصل سوم: مکانیک درشت ساختاری

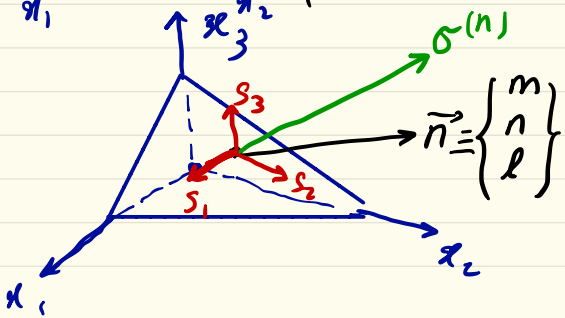
تنش:







$$\sigma \equiv \sigma_{ij} \equiv \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \rightsquigarrow \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{13} \end{Bmatrix}$$

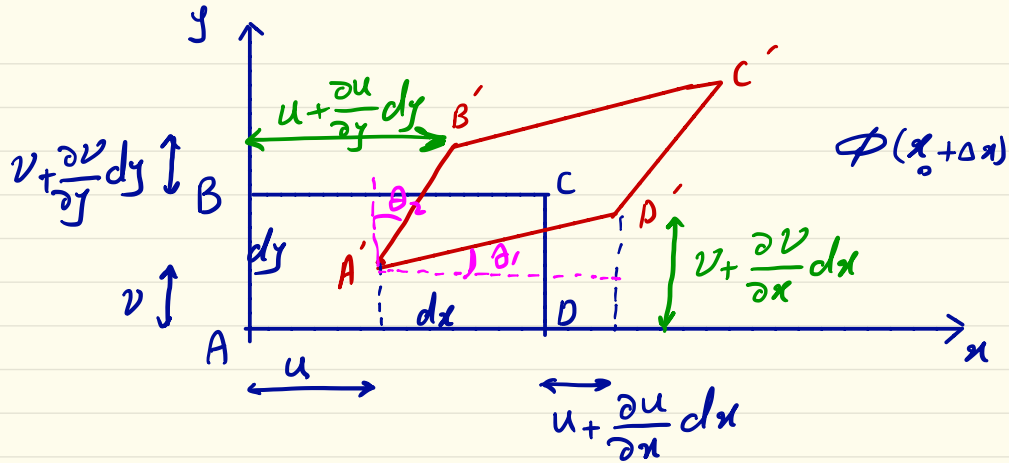


$$\sigma^{(n)} = \begin{Bmatrix} S_1 \\ S_2 \\ S_3 \end{Bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots \\ \vdots & \ddots & \end{bmatrix} \begin{Bmatrix} m \\ n \\ \ell \end{Bmatrix}$$

$$\boxed{\sigma_i^{(n)} = \sigma_{ij}\, n_j}$$

کرنش:

$$\phi(x_0+\Delta x)=\phi(x_0)+\frac{d\phi}{dx}\Delta x+\cdots$$



$$\varepsilon_x=\frac{\Delta\ell}{\ell}=\frac{\frac{\partial u}{\partial x}dx}{dx}=\frac{\partial u}{\partial x}$$

$$\varepsilon_y=\frac{\Delta\ell}{\ell}=\frac{\frac{\partial v}{\partial y}dy}{dy}=\frac{\partial v}{\partial y}$$

$$\gamma_{xy}=\frac{\frac{\partial v}{\partial x}dx}{dx}+\frac{\frac{\partial u}{\partial y}dy}{dy}=\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\quad\rightsquigarrow\quad\varepsilon_{xy}=\frac{1}{2}\gamma_{xy}$$

$$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right)$$

روابط تنش و کرنش:

$$\varepsilon_{xx} = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\varepsilon_{yy} = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\varepsilon_{zz} = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{xz} = \frac{\tau_{xz}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$G = \frac{E}{2(1+\nu)}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}$$

$$\{\varepsilon\} = [S]\{\sigma\}$$

نرمی

$$\{\sigma\} = [C]\{\varepsilon\}$$

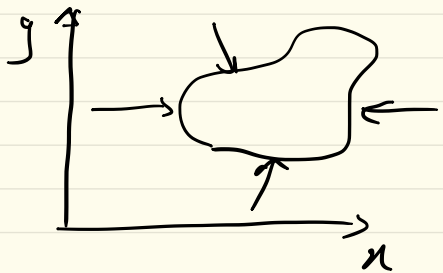
ماتریس سفتی

$$[S] = [C]^{-1}$$

مسائل تنش صفحه‌ای و کرنش صفحه‌ای:

تنش صفحه‌ای:

ابعاد در جهت محکم باشد، حوالی قطعه در جهت z تغییر نکند، بارگذاری در جهت z نداشته باشیم



$$\sigma_3 = 0 = \sigma_{31} = \sigma_{32}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

کرنش صفحه‌ای:

طول در جهت z زیاد باشد یا بارگذاری در جهت z نداشته باشیم.

$$\varepsilon_3 = 0 = \varepsilon_{31} = \varepsilon_{32}$$

$$[S] = \begin{bmatrix} \lambda+2G & \lambda & 0 \\ \lambda & \lambda+2G & 0 \\ 0 & 0 & G \end{bmatrix}$$

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

غشائی:

ابعاد در جهت 3 خلی کم است.

$\sigma_3 = 0 = \sigma_{31} = \sigma_{32}$

$\varepsilon_3 \approx$ قابل صرف نظر کردن است