بسم الله الرحمن الرحیم   مواد مرکب   جلسه ۸

۱- ارتباط تنش و کرنش برای مواد غیرایزوتروپ:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{16} \\ S_{12} & S_{22} & S_{26} \\ S_{16} & S_{26} & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

$$\varepsilon_2 = S_{12}\sigma_1 + S_{22}\sigma_2 + S_{26}\tau_{12}$$

$$\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1}$$

هنگامی که فقط $\sigma = \sigma_1$ اعمال شود.

$$\frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j}$$

$\eta_{i,ij}$: ضریب اثر متقابل اول Leknitski

$$\eta_{i,ij} = \frac{\varepsilon_i}{\gamma_{ij}}$$

وقتی که فقط $\sigma = \tau_{ij}$ اعمال شود.

$\eta_{ij,i}$: ضریب اثر متقابل دوم Leknitski

$$\eta_{ij,k} = \frac{\gamma_{ij}}{\varepsilon_k}$$

وقتی که فقط $\sigma = \sigma_k$ اعمال شود.

$\mu_{ij,kl}$: ضریب اثر متقابل chentsov

$$\mu_{ij,kl} = \frac{\gamma_{ij}}{\gamma_{kl}}$$

وقتی که فقط $\sigma = \tau_{kl}$ اعمال شود.

S: متقارن $\Longrightarrow$

$$\begin{cases} \dfrac{\mu_{ij,kl}}{G_{kl}} = \dfrac{\mu_{kl,ij}}{G_{ij}} \\[2ex] \dfrac{\eta_{ij,i}}{E_i} = \dfrac{\eta_{i,ij}}{G_{ij}} \end{cases}$$

$$\gamma_{13} = \frac{\eta_{13,1}}{E_1}\sigma_1 + \frac{\eta_{13,2}}{E_2}\sigma_2 + \frac{\mu_{13,12}}{G_{12}}\tau_{12} = \frac{\eta_{1,13}\sigma_1 + \eta_{2,13}\sigma_2 + \mu_{12,13}\tau_{12}}{G_{13}}$$

- ثابت‌های مهندسی در جهت غیراصلی برای مواد ارتوتروپ

E_x ، E_y ، ν_{xy} ، G_{xy} ، $\eta_{xy,x}$ و $\eta_{xy,y}$

$$E_x = \frac{\varepsilon_x}{\sigma_x}$$

اگر فقط $\sigma = \sigma_x$ اعمال شود.

$$\frac{1}{E_X} = \frac{1}{E_1}cos^4\theta + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)sin^2\theta cos^2\theta + \frac{1}{E_2}sin^4\theta$$

$$\frac{1}{E_Y} = \frac{1}{E_1}sin^4\theta + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)sin^2\theta cos^2\theta + \frac{1}{E_2}cos^4\theta$$

$$\nu_{XY} = E_X\left[\frac{\nu_{12}}{E_1}(sin^4\theta + cos^4\theta) - \left(\frac{1}{E_1} + \frac{1}{E_2} - \frac{1}{G_{12}}\right)sin^2\theta cos^2\theta\right]$$

$$\frac{1}{G_{XY}} = 2\left(\frac{2}{E_1} + \frac{2}{E_2} + \frac{4\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)sin^2\theta cos^2\theta + \frac{1}{G_{12}}(sin^4\theta + cos^4\theta)$$

$$\eta_{XY,X} = E_X\left[\left(\frac{2}{E_1} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)sin\theta cos^3\theta - \left(\frac{2}{E_2} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)sin^3\theta cos\theta\right]$$

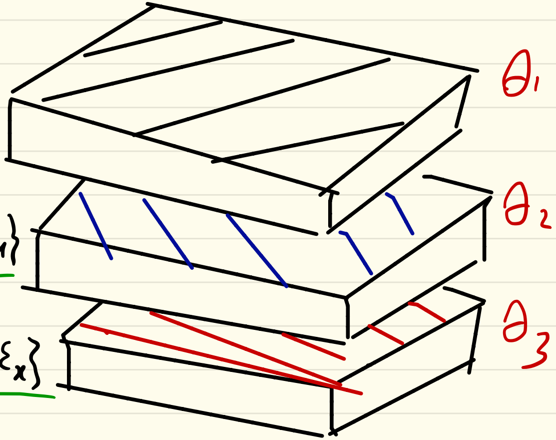
$$\eta_{XY,Y} = E_Y\left[\left(\frac{2}{E_1} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)sin^3\theta cos\theta - \left(\frac{2}{E_2} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)sin\theta cos^3\theta\right]$$

۳-۳ - رفتار مواد مرکب چندلایه : (لمینیت)

$\{\sigma_x\} = [\bar{Q}]_1 \{\varepsilon_x\}$

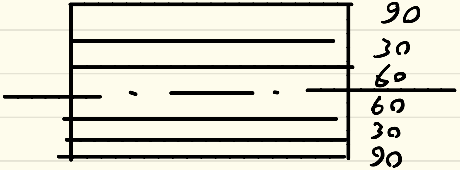
$\{\sigma_x\} = [\bar{Q}]_2 \{\varepsilon_x\}$

$\{\sigma_x\} = [\bar{Q}]_3 \{\varepsilon_x\}$

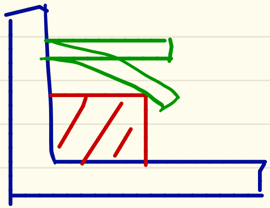


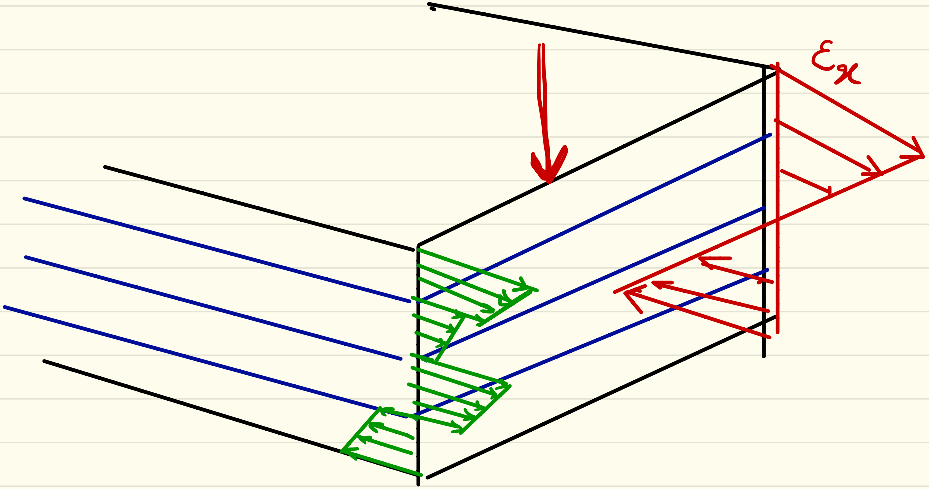
$[\theta_1, \theta_2, \theta_3]$

$\theta_1 / \theta_2 / \theta_3$

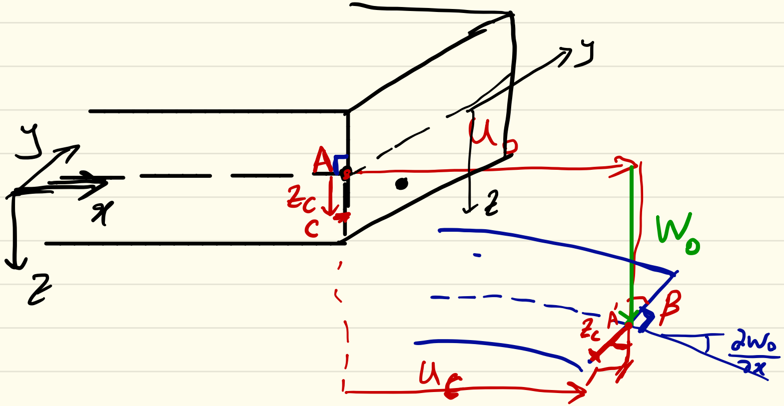


$[90, 30, 60]_S$

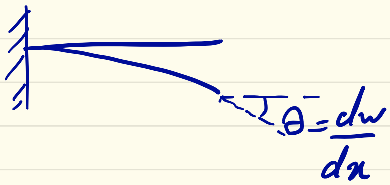




$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} \quad \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$



$$u_c = u_0 - z_c\,\beta \quad , \beta = \frac{\partial w_0}{\partial x}$$



$$u(x,y,z) = u_0 - z\,\frac{\partial w_0}{\partial x} \quad , \quad v = v_0 - z\,\frac{\partial w_0}{\partial y}$$

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\,\frac{\partial^2 w_0}{\partial x^2}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} - z\,\frac{\partial^2 w_0}{\partial y^2}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right) - 2z\,\frac{\partial^2 w_0}{\partial x\,\partial y}$$

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}$$

کرنش لایه میانی

$$\begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}$$

انحناهای لایه میانی

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix}$$

$$\boxed{\{\varepsilon\} = \{\varepsilon\}^0 + z\{k\}}$$