بسم الله الرحمن الرحیم

مواد مرکب — جلسه ۷

۳ ـ ۲ ـ رفتار مکانیکی اجسام تک لایه:

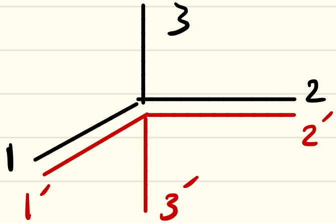
در حالت کلی

$$\{\sigma\} = [C]\{\varepsilon\}$$

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} \qquad i,j,k,l = 1,2,3$$

$$\sigma_i = C_{ij}\,\varepsilon_j \qquad i,j = 1,\dots,6$$

21

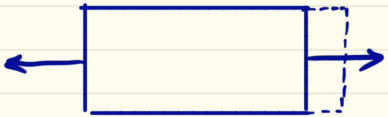


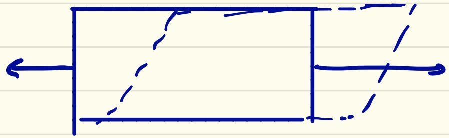
حالت‌های خاص مواد:

۱ـ جسم یک صفحه تقارن داشته باشد:

$$[C] = [C]'$$

$$
\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix}
=
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\
C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\
C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\
0 & 0 & 0 & C_{44} & C_{45} & 0 \\
0 & 0 & 0 & C_{45} & C_{55} & 0 \\
C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66}
\end{bmatrix}
\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix}
\longrightarrow 13
$$



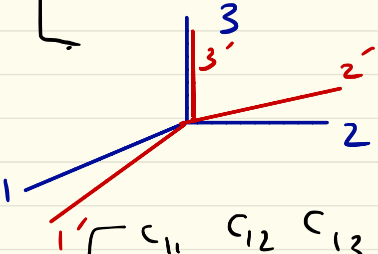


$$
\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \gamma_{12} \end{Bmatrix}
=
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\
S_{12} & S_{22} & S_{23} & 0 & 0 & S_{26} \\
S_{13} & & & & & \\
0 & & & & & \\
0 & & & \ddots & & \\
S_{16} & & & & & S_{66}
\end{bmatrix}
\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \vdots \\ \tau_{12} \end{Bmatrix}
$$

۲ـ دو صفحه تقارن ← ارتوتروپ

$$
\{\sigma\} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ & C_{22} & C_{23} & 0 & 0 & 0 \\ & & C_{33} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ & \text{Sym} & & & C_{55} & 0 \\ & & & & & C_{66} \end{bmatrix} \cdot \{\varepsilon\} \rightarrow 9
$$

۳ـ یک صفحه ایزوتروپ ← ایزوتروپ عرضی



$$
\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ & C_{11} & C_{13} & 0 & 0 & 0 \\ & & C_{33} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ & & & & C_{44} & 0 \\ & & & & & \left(\frac{C_{11}-C_{12}}{2}\right) \end{bmatrix} \sim 5
$$

4_ دو صفحه ایزوتروپ ← بینهایت صفحه ایزوتروپ ← ایزوتروپ

$$\begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ & C_{11} & C_{12} & 0 & 0 & 0 \\ & & C_{11} & 0 & 0 & 0 \\ & & & \frac{C_{11}-C_{12}}{2} & 0 & 0 \\ & & & & \frac{C_{11}-C_{12}}{2} & 0 \\ & & & & & \frac{C_{11}-C_{12}}{2} \end{bmatrix} \rightarrow \boxed{2}$$

۳-۲-۲- ثابت‌های مهندسی مواد ارتوتروپ:

$$
[S_{ij}] = \begin{bmatrix}
\frac{1}{E} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\
-\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\
-\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}}
\end{bmatrix}
$$

(19)

$$\frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1}$$

$$S_{ij} = S_{ji} \Rightarrow \frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j}$$

$$\{\varepsilon\} = [S]\{\sigma\}$$

$$\{\sigma\} = [C]\{\varepsilon\}$$

$$C_{11} = \frac{S_{22}S_{33}-S_{23}{}^2}{S}$$

$$C_{12} = \frac{S_{13}S_{23}-S_{12}S_{33}}{S}$$

$$C_{22} = \frac{S_{33}S_{11}-S_{13}{}^2}{S}$$

$$C_{13} = \frac{S_{12}S_{23}-S_{13}S_{22}}{S}$$

$$C_{33} = \frac{S_{11}S_{22}-S_{12}{}^2}{S}$$

$$C_{23} = \frac{S_{12}S_{13}-S_{23}S_{11}}{S}$$

$$C_{44} = \frac{1}{S_{44}}, \quad C_{55} = \frac{1}{S_{55}}, \quad C_{66} = \frac{1}{S_{66}}$$

$$S = S_{11}\, S_{22}\, S_{33} - S_{11}\, S_{23}^2 - S_{22}\, S_{13}^2 - S_{33}\, S_{12}^2 + 2S_{12}\, S_{23}\, S_{13}$$

ماتریس سختی براساس پارامترهای مهندسی :

$$C_{11} = \frac{1-\nu_{23}\nu_{32}}{E_2E_3\Delta}, \quad C_{22} = \frac{1-\nu_{13}\nu_{31}}{E_1E_3\Delta}, \quad C_{33} = \frac{1-\nu_{12}\nu_{21}}{E_1E_2\Delta}$$

$$C_{12} = \frac{\nu_{21}+\nu_{31}\nu_{23}}{E_2E_3\Delta} = \frac{\nu_{12}+\nu_{32}\nu_{13}}{E_1E_3\Delta}$$

$$C_{13} = \frac{\nu_{31}+\nu_{21}\nu_{32}}{E_2E_3\Delta} = \frac{\nu_{13}+\nu_{12}\nu_{23}}{E_1E_2\Delta}$$

$$C_{23} = \frac{\nu_{32}+\nu_{12}\nu_{31}}{E_1E_3\Delta} = \frac{\nu_{23}+\nu_{21}\nu_{13}}{E_1E_2\Delta}$$

$$C_{44} = G_{23}, \quad C_{55} = G_{31}, \quad C_{66} = G_{16}$$

$$\Delta = \frac{1-\nu_{12}\nu_{21}-\nu_{23}\nu_{32}-\nu_{31}\nu_{13}-2(\nu_{21}\nu_{32}\nu_{13})}{E_1E_2E_3}$$

3_2_3 ـ ارتباط بین پارامترهای مهندسی:

الف ـ مواد ایزوتروپ:

$$\varepsilon = \frac{\sigma}{E}$$

$$G = \frac{E}{2(1+\nu)}$$

$$E, G > 0 \implies -1 < \nu$$

$$\theta = \frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{P}{K}$$

فشار هیدرواستاتیک

مدول بالک

$$K = \frac{E}{3(1-2\nu)}$$

$$K > 0 \implies \nu < 1/2$$

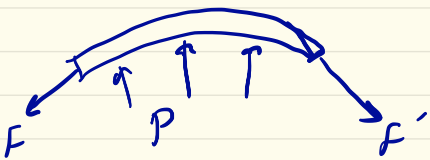
$$\implies -1 < \nu < 1/2$$

ب ـ مواد ارتوتروپ

$$S_{ij} = S_{ji} \Rightarrow \frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j}$$

$$S_{11}, S_{22}, S_{33}, S_{44}, S_{55}, S_{66} > 0 \Rightarrow E_1, E_2, E_3, G_{12}, G_{13}, G_{23} > 0$$

3-2-4- ارتباط تنش و کرنش براساس تنش غشائی (مواد ارتوتروپ)



$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

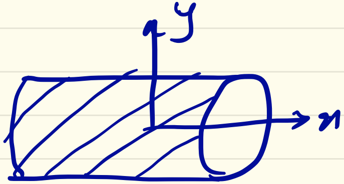
$$S_{11} = \frac{1}{E_1}, \quad S_{22} = \frac{1}{E_2}, \quad S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2}, \quad S_{66} = \frac{1}{G_{12}}$$

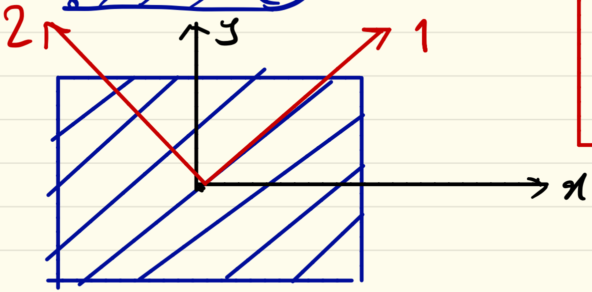
$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix}$$

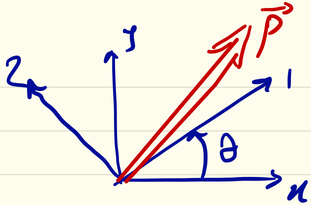
$$\{\sigma\} = [Q]\,\{\varepsilon\}$$

- ارتباط تنش و کرنش در مواد ارتوتروپ در جهت دلخواه

$$\boxed{\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = [\bar{Q}]\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}}$$







$$[T(\theta)] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

ε

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = [T(\theta)] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{\gamma_{12}}{2} \end{Bmatrix} = [T(\theta)] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}/2 \end{Bmatrix}$$

$$T(-\theta) = T^{-1}(\theta)$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [R] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}/2 \end{Bmatrix} , \quad [R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

$$\hookrightarrow [Q] \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

$$\hookrightarrow [R] \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{Bmatrix}$$

$$\hookrightarrow [T] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}/2 \end{Bmatrix}$$

$$[R]^{-1} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\{\sigma_x\} = \overbrace{[T]^{-1}[Q]\underbrace{[R][T][R]^{-1}}_{[R][T][R]^{-1} = [T]^{-T}}}^{[\bar{Q}]} \{\varepsilon_x\}$$

$$\boxed{[\bar{Q}] = [T]^{-1}[Q][T]^{-T}}$$

$$\bar{Q}_{11} = Q_{11}cos^4\theta + 2(Q_{12} + 2Q_{66})sin^2\theta cos^2\theta + Q_{22}sin^4\theta$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})sin^2\theta cos^2\theta + Q_{12}(sin^4\theta + cos^4\theta)$$

$$\bar{Q}_{22} = Q_{11}sin^4\theta + 2(Q_{12} + 2Q_{66})sin^2\theta cos^2\theta + Q_{22}cos^4\theta$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})\sin\theta\, cos^3\theta + (Q_{12} - Q_{22} + 2Q_{66})sin^3\theta\cos\theta$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})sin^3\theta\cos\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin\theta\, cos^3\theta$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})sin^2\theta cos^2\theta + Q_{66}(sin^4\theta + cos^4\theta)$$

$$\overline{S_{12}} = (S_{11} + S_{22} - S_{66})sin^2\theta cos^2\theta + S_{12}(sin^4\theta + cos^4\theta)$$

$$\overline{S_{11}} = S_{11}cos^4\theta + (2S_{12} + S_{66})sin^2\theta cos^2\theta + S_{22}sin^4\theta$$

$$\overline{S_{22}} = S_{11}sin^4\theta + (2S_{12} + S_{66})sin^2\theta cos^2\theta + S_{22}cos^4\theta$$

$$\overline{S_{16}} = (2S_{11} - 2S_{12} - S_{66})\sin\theta\, cos^3\theta - (2S_{22} - 2S_{12} - S_{66})sin^3\theta\cos\theta$$

$$\overline{S_{26}} = (2S_{11} - 2S_{12} - S_{66})sin^3\theta\cos\theta - (2S_{22} - 2S_{12} - S_{66})\sin\theta\, cos^3\theta$$

$$\overline{S_{66}} = 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})sin^2\theta cos^2\theta + S_{66}(sin^4\theta + cos^4\theta)$$