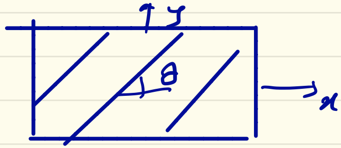
بسم الله الرحمن الرحیم — مواد مرکب — جلسه ۱۱

۳-۳- ثابت های مهندسی برای صفحات چندلایه :

$E_x, E_y, ..$

الف - برای لمینیت تک لایه



$$E_x = \frac{E_1}{m^4 + \left(\frac{E_1}{G_{12}} - 2\nu_{12}\right) n^2 m^2 + \frac{E_1}{E_2} n^4}$$

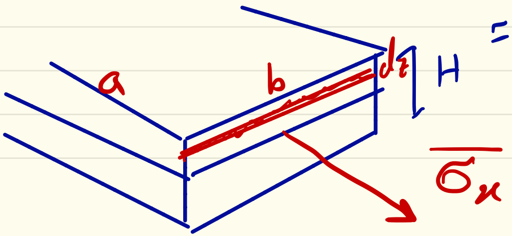
$m, n = \cos\theta, \sin\theta$

$\{\varepsilon\} = [S]\{\sigma\}$

$$\eta_{xy,y} = \frac{\gamma_{xy}}{\varepsilon_y}$$ در حالتی که فقط $\sigma_y \neq 0$ $$= \frac{\bar{S}_{26}}{\bar{S}_{22}}$$

$$\eta_{x,xy} = \frac{\varepsilon_x}{\gamma_{xy}}$$ در حالتی که فقط $\tau_{xy} \neq 0$ $$= \frac{\bar{S}_{16}}{\bar{S}_{66}}$$

ب - برای لمینیت چندلایه



$$\bar{\sigma}_x = \frac{1}{H}\int_{-H/2}^{+H/2} \sigma_x \, dz \quad , \quad \bar{\sigma}_y = \frac{1}{H}\int_{-H/2}^{+H/2} \sigma_y \, dz \quad , \quad \bar{\tau}_{xy} = \frac{1}{H}\int_{-H/2}^{+H/2} \tau_{xy} \, dz$$

$$\bar{\sigma}_x = \frac{1}{H} N_x \quad , \quad \bar{\sigma}_y = \frac{1}{H} N_y \quad , \quad \bar{\tau}_{xy} = \frac{1}{H} N_{xy}$$

$$\{N\} = [A]\{\varepsilon\}^{0} + [B]\{K\} \xrightarrow{\text{اگر } [B]=0} \{N\} = [A]\{\varepsilon\}^{0} \Rightarrow \{\varepsilon\}^{0} = [A]^{-1}\{N\}$$

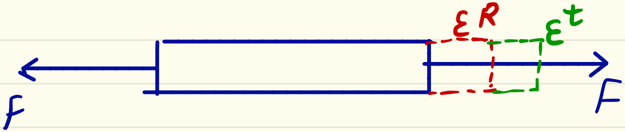
$$\{\varepsilon\}^{0} = [A]^{-1} H \{\bar{\sigma}\}$$

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \underbrace{\begin{bmatrix} a_{11}H & a_{12}H & 0 \\ a_{12}H & a_{22}H & 0 \\ 0 & 0 & a_{66}H \end{bmatrix}}_{\text{ماتریس نرمی موثر برای لمینیت}} \begin{Bmatrix} \bar{\sigma}_x \\ \bar{\sigma}_y \\ \bar{\tau}_{xy} \end{Bmatrix}$$

$$\bar{E}_x = \frac{1}{a_{11}H} \quad , \quad \bar{E}_y = \frac{1}{a_{22}H} \quad , \quad \bar{G}_{xy} = \frac{1}{a_{66}H}$$

$$\bar{\nu}_{xy} = -\frac{a_{12}}{a_{11}} \quad , \quad \bar{\nu}_{yx} = -\frac{a_{12}}{a_{22}}$$

۳-۴- آنالیز تنش صفحات چندلایه با در نظر گرفتن حرارت



$$\varepsilon = \varepsilon^R + \varepsilon^t$$

$$\Delta^t = \alpha l \Delta t \quad , \quad \varepsilon^t = \alpha \Delta t$$

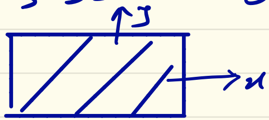
α : ضریب انبساط حرارتی

$$\sigma_x = E_x(\varepsilon - \varepsilon^t)$$

$$\{\sigma\} = [C]\left(\{\varepsilon\} - \{\varepsilon\}^t\right) \quad , \quad \{\varepsilon\}^t = \{\alpha\}\Delta t$$

برای مواد ارتوتروپ در جهات اصلی داریم:

$$\{\alpha\} = \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix}$$



$$\boxed{\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \frac{1}{2}\alpha_{xy} \end{Bmatrix} = [T(\theta)]^{-1} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix}}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = [Q] \begin{Bmatrix} \varepsilon_1 - \alpha_1 \Delta t \\ \varepsilon_2 - \alpha_2 \Delta t \\ \gamma_{12} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = [\bar{Q}]_k \begin{Bmatrix} \varepsilon_x - \alpha_x \Delta t \\ \varepsilon_y - \alpha_y \Delta t \\ \gamma_{xy} - \alpha_{xy} \Delta t \end{Bmatrix}_k$$

$$\{N\} = [A]\{\varepsilon\}^0 + [B]\{K\} - \begin{Bmatrix} N_x^t \\ N_y^t \\ N_{xy}^t \end{Bmatrix}$$

$$\begin{Bmatrix} N_x^t \\ N_y^t \\ N_{xy}^t \end{Bmatrix} = \sum \int_{z_{k-1}}^{z_k} [\bar{Q}]_k \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k \Delta t \, dz = \sum_{k=1}^{N} [\bar{Q}]_k \{\alpha\}_k \Delta t (z_k - z_{k-1})$$

$$\{M\} = [B]\{\varepsilon\}^0 + [D]\{K\} - \{M\}^t$$

$$\begin{Bmatrix} M_x^t \\ M_y^t \\ M_{xy}^t \end{Bmatrix} = \sum \int [\bar{Q}]\{\alpha\} z \Delta t \, dz = \sum_{k=1}^{N} [\bar{Q}]_k \{\alpha\}_k \Delta t \tfrac{1}{2}(z_k^2 - z_{k-1}^2)$$

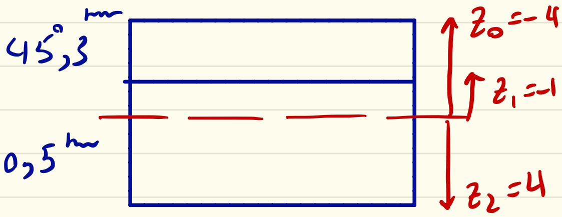
$$\left\{\begin{array}{c}\bar{N}\\ \hdashline \bar{M}\end{array}\right\} = \left[\begin{array}{c:c} A & B \\ \hdashline B & D \end{array}\right] \left\{\begin{array}{c}\varepsilon^{\circ}\\ \hdashline K\end{array}\right\}$$

$$\{\bar{N}\} = \{N\} + \{N\}^{t}$$

$$\{\bar{M}\} = \{M\} + \{M\}^{t}$$

$$\left\{\begin{array}{c}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{array}\right\}_k = [\bar{Q}]_k \left(\{\varepsilon\}^{\circ} + z\{K\} - \{\alpha\}\Delta t\right)$$

مثال ۶: تنش‌های پسماند را برای مثال (۱) براساس تغییرات درجه حرارت در زمان ساخت 125° و درجه حرارت محیط 25° محاسبه کنید.



$$\alpha_1 = 7\times10^{-6} \ \ {}^1/_{\circ C}$$

$$\alpha_2 = 23\times10^{-6} \ \ {}^1/_{\circ C}$$

$$\begin{Bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{Bmatrix}_0 = \begin{Bmatrix}\alpha_1\\ \alpha_2\\ 0\end{Bmatrix} = \begin{Bmatrix}7\\ 23\\ 0\end{Bmatrix}\times10^{-6} \quad {}^1/_{\circ C}$$

$$\begin{Bmatrix}\alpha_x\\ \alpha_y\\ \frac{1}{2}\alpha_{xy}\end{Bmatrix}_{45^\circ} = \left[T(45^\circ)\right]^{-1} \begin{Bmatrix}7\\ 23\\ 0\end{Bmatrix}\times10^{-6}$$

$$\begin{Bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{Bmatrix}_{45^\circ} = \begin{Bmatrix}15\\ 15\\ -16\end{Bmatrix}10^{-6} \quad {}^1/_{\circ C}$$

$$\Delta t = 25 - 125 = -100$$

$$[\bar{Q}]_0 \{\alpha\}_0 \Delta t = \begin{Bmatrix} -15.6 \\ -5.09 \\ 0 \end{Bmatrix} 10^{-3}$$

$$[\bar{Q}]_{45^\circ} \{\alpha\}_{45^\circ} \Delta t = \begin{Bmatrix} -10.35 \\ -10.35 \\ -5.26 \end{Bmatrix} 10^{-3}$$

$$\{N\}^t = \sum \underbrace{[\bar{Q}]_k \{\alpha\}_k \Delta t}\,(z_k - z_{k-1}) = \begin{Bmatrix} -15.61 \\ -5.09 \\ 0 \end{Bmatrix} \times 10^{-3} \left[(4)-(-1)\right] + \begin{Bmatrix} -10.35 \\ -10.35 \\ -5.26 \end{Bmatrix} 10^{-3} \left[(-1) - (-4)\right]$$

$$= \begin{Bmatrix} -109.1 \\ -56.5 \\ -15.78 \end{Bmatrix} \times 10^{-3} \qquad \frac{GN\text{-}mm}{m^2}$$

$$\{M\}^t = \sum \underbrace{[\bar{Q}]_k \{\alpha\}_k \Delta t}\, \tfrac{1}{2}(z_k^2 - z_{k-1}^2) = \begin{Bmatrix} -15.61 \\ -5.09 \\ 0 \end{Bmatrix} 10^{-3} \times \frac{1}{2} \left[(4)^2 - (-1)^2\right]$$

$$+ \begin{Bmatrix} -10.35 \\ -10.35 \\ -5.62 \end{Bmatrix} 10^{-3} \times \tfrac{1}{2} \left[(-1)^2 - (-4)^2\right]$$

$$= \begin{Bmatrix} -39.45 \\ 39.45 \\ 39.45 \end{Bmatrix} \times 10^{-3} \quad \frac{GN\text{-}mm}{m^2}$$

$$\{\bar{N}\} = \{N\} + \{N\}^t = \{0\} + \{N\}^t$$

$$\{\bar{M}\} = \{M\} + \{M\}^t = \{0\} + \{M\}^t$$

$$\begin{Bmatrix} \bar{N} \\ \hline \bar{M} \end{Bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline B & D \end{array}\right] \begin{Bmatrix} \varepsilon^\circ \\ \hline K \end{Bmatrix} \Longrightarrow \begin{Bmatrix} \varepsilon^\circ \\ \hline K \end{Bmatrix} = \left[\begin{array}{c|c} A' & B' \\ \hline C' & D' \end{array}\right] \begin{Bmatrix} \bar{N} \\ \hline \bar{M} \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} \varepsilon_x^\circ \\ \varepsilon_y^\circ \\ \gamma_{xy}^\circ \end{Bmatrix} = \begin{Bmatrix} -8.14 \\ -20.2 \\ 6.99 \end{Bmatrix} \times 10^{-4}$$

$$\begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix} = \begin{Bmatrix} 0.58 \\ -1 \\ -2.55 \end{Bmatrix} \times 10^{-4}$$

برای یافتن تنش در لایه‌های مختلف داریم:

$$\{\varepsilon\}^R = \{\varepsilon\} - \{\varepsilon\}^t = \begin{Bmatrix} \varepsilon_x^\circ + zK_x \\ \varepsilon_y^\circ + zK_y \\ \gamma_{xy}^\circ + zK_{xy} \end{Bmatrix} - \begin{Bmatrix} \alpha_x \Delta t \\ \alpha_y \Delta t \\ \alpha_{xy} \Delta t \end{Bmatrix}$$

$$\{\sigma\}_k = [\bar{Q}]_k \{\varepsilon\}^R$$

$$\begin{Bmatrix} \varepsilon_x^R \\ \varepsilon_y^R \\ \gamma_{xy}^R \end{Bmatrix}_{0^\circ} = \begin{Bmatrix} -13+0.58\,z \\ 2.8-1.0\,z \\ 6.99-2.35\,z \end{Bmatrix} \times 10^{-4} \quad , \quad \begin{Bmatrix} \varepsilon_x^R \\ \varepsilon_y^R \\ \gamma_{xy}^R \end{Bmatrix}_{45^\circ} = \begin{Bmatrix} 6.86+0.58\,z \\ -5.2-1.0\,z \\ -9.01-2.33\,z \end{Bmatrix} \times 10^{-4}$$

برای لایه صفر درجه

$z=4$ $$\{\sigma\}_0 = \begin{bmatrix} 20 & 0.7 & 0 \\ & 0.2 & 0 \\ sym & & 0.7 \end{bmatrix} \begin{Bmatrix} 1.18 \\ -1.2 \\ -2.4 \end{Bmatrix} 10^{-4} \quad GN/m^2$$

$z=-1$ $$\{\sigma\}_0 = [\quad Q \quad] \begin{Bmatrix} -1.72 \\ 3.8 \\ 9.34 \end{Bmatrix} \quad GN/m^2$$

برای لایه 45°

$z=-1$ $$\{\sigma\}_{45^\circ} = \begin{Bmatrix} -1.03 \\ -2.51 \\ -2.49 \end{Bmatrix} \times 10^{-3} \quad GN/m^2$$

$z=-4$ $$\{\sigma\}_{45^\circ} = \begin{Bmatrix} -4.27 \\ 0.71 \\ -0.75 \end{Bmatrix} \times 10^{-3} \quad GN/m^2$$