جلسه ۲ — مکانیک رشد — بسم الله الرحمن الرحیم

@utgrowth

$$\vec{R} = |\vec{R}|\,\vec{e}_1{}' = |\vec{R}|\cos(\vec{e}_1,\vec{e}_1{}')\,\vec{e}_1 + |\vec{R}|\cos(\vec{e}_2,\vec{e}_1{}')\,\vec{e}_2 + |\vec{R}|\cos(\vec{e}_3,\vec{e}_1{}')\,\vec{e}_3$$

$$\Rightarrow \quad \vec{e}_1{}' = \cos(\vec{e}_1,\vec{e}_1{}')\,\vec{e}_1 + \cos(\vec{e}_2,\vec{e}_1{}')\,\vec{e}_2 + \cos(\vec{e}_3,\vec{e}_1{}')\,\vec{e}_3$$

$$a_{ij} = \cos(\vec{e}_i{}',\vec{e}_j)$$

$$\vec{e}_1{}' = a_{1j}\,\vec{e}_j \quad \Rightarrow \quad \boxed{\vec{e}_i{}' = a_{ij}\,\vec{e}_j} = a_{ik}\,\vec{e}_k$$

$$\vec{e}_1 = \cos(\vec{e}_1,\vec{e}_1{}')\,\vec{e}_1{}' + \cos(\vec{e}_1,\vec{e}_2{}')\,\vec{e}_2{}' + \cos(\vec{e}_1,\vec{e}_3{}')\,\vec{e}_3{}'$$

$$= a_{11}\,\vec{e}_1{}' + a_{21}\,\vec{e}_2{}' + a_{31}\,\vec{e}_3{}'$$

$$\boxed{\vec{e}_i = a_{ji}\,\vec{e}_j{}'}$$

$$\vec{e}_i\,' \cdot \vec{e}_j = a_{ik}\vec{e}_k \cdot \vec{e}_j = a_{ik}\,\delta_{kj} = a_{ij}$$

$$\boxed{a_{ij} = \vec{e}_i\,'' \cdot \vec{e}_j = \vec{e}_j \cdot \vec{e}_i\,'}$$

$$\vec{e}_i \cdot \vec{e}_j = \delta_{ij}$$

$$\vec{e}_i\,' \cdot \vec{e}_j\,' = \delta_{ij} = (a_{ik}\vec{e}_k) \cdot (a_{j\ell}\,\vec{e}_\ell) = a_{ik}\,a_{j\ell}\,(\underbrace{\vec{e}_k \cdot \vec{e}_\ell}_{\delta_{k\ell}})$$

$$\boxed{a_{ik}\,a_{jk} = \delta_{ij}}$$

$$\vec{A} = A_j'\,\vec{e}_j\,' = A_i\,\vec{e}_i$$

$$\vec{F} = m\vec{a}$$

$$\begin{cases} F_i = m a_i \\ F_i' = m a_i' \end{cases}$$

$$A'_j \vec{e}'_j = A_i \vec{e}_i = A_i\, a_{ji}\, \vec{e}'_j \Longrightarrow (A'_j - A_i a_{ji})\vec{e}'_j = \vec{0}$$

$$\boxed{A'_j = a_{ji} A_i}$$

Transformation Law

first-order tensor

$$T'_{ij} = a_{ik} a_{jl} T_{kl}$$

$$T'_{\underbrace{ijk\ldots}_{\text{N اندیس}}} = \underbrace{a_{im} a_{jn} \ldots\ldots}_{\text{N تا}} T_{mn\ldots}$$

Quotient Law (Rules)

A_i = a first-order tensor

B_{ij} = a second-order tensor

$\Rightarrow \overbrace{A_i B_{jk}}^{D_{ijk}}$: a third-order tensor

$$\left.\begin{aligned} A_i' &= a_{ip} A_p \\ B_{jk}' &= a_{jq}\, a_{kr}\, B_{qr} \end{aligned}\right\} \Rightarrow \boxed{A_i' B_{jk}' = a_{ip}\, a_{jq}\, a_{kr}\, A_p B_{qr}}$$

$$\left.\begin{aligned} &A_{ijk\ldots} \text{ is an } N^{\underline{th}}\text{ order tensor} \\ &B_{rst\ldots} \text{ is an } M^{\underline{th}}\text{ order tensor} \end{aligned}\right\} \Rightarrow A_{ijk\ldots} B_{rst\ldots} \left(\equiv C_{ijk\ldots rst\ldots}\right)$$

is an $(N+M)^{\underline{th}}$ order tensor

Contraction Law

$$B_{ijkl} \rightsquigarrow \underbrace{B_{ijki}}_{2^{\underline{nd}}\text{ order}} \equiv B_{ljkl}$$

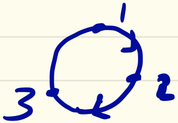
1.4 - Permutation Symbol

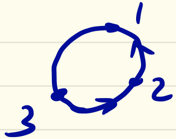
$$\vec{e}_1 \times \vec{e}_1 = \vec{e}_2 \times \vec{e}_2 = \vec{e}_3 \times \vec{e}_3 = 0$$

$$\vec{e}_1 \times \vec{e}_2 = \vec{e}_3 \,,\quad \vec{e}_2 \times \vec{e}_3 = \vec{e}_1 \,,\quad \vec{e}_3 \times \vec{e}_1 = e_2$$

$$\vec{e}_2 \times \vec{e}_1 = -\vec{e}_3 \,,\quad \vec{e}_3 \times \vec{e}_2 = -\vec{e}_1 \,,\quad \vec{e}_1 \times \vec{e}_3 = -\vec{e}_2$$

$$\begin{cases} \varepsilon_{ijk}=0 & \text{if two or all indeces are the same} \\ \varepsilon_{123}=\varepsilon_{231}=\varepsilon_{312}=1 & \\ \varepsilon_{132}=\varepsilon_{321}=\varepsilon_{213}=-1 & \end{cases}$$





$$\boxed{\vec{e}_i \times \vec{e}_j = \varepsilon_{ijk}\,\vec{e}_k}$$

Problem: $\vec{A}\cdot\vec{B}=?$

$$\vec{A}\cdot\vec{B}=(a_i\vec{e}_i)\cdot(b_j\vec{e}_j)=a_ib_j\underbrace{(\vec{e}_i\cdot\vec{e}_j)}_{\delta_{ij}}=a_ib_i=a_jb_j$$

Problem: $\vec{A}\times\vec{B}=?$

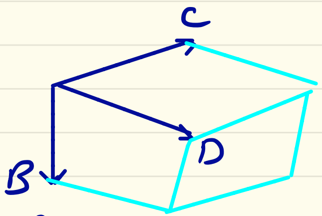
$$\vec{A}\times\vec{B}=(A_i\vec{e}_i)\times(B_j\vec{e}_j)=A_iB_j(\vec{e}_i\times\vec{e}_j)=A_iB_j\,\varepsilon_{ijk}\,\vec{e}_k$$

Problem: $\vec{B}\times\vec{C}\cdot\vec{D} = ?$

$$\vec{B}\times\vec{C}\cdot\vec{D} = (B_i c_j \varepsilon_{ijk}\,\vec{e}_k)\cdot(D_\ell\,\vec{e}_\ell) = B_i c_j D_\ell\,\varepsilon_{ijk}\underbrace{(\vec{e}_k\cdot\vec{e}_\ell)}_{\delta_{k\ell}}$$

$$= B_i c_j D_\ell\,\varepsilon_{ij\ell}$$

$\vec{B}\times\vec{C}\cdot\vec{D}$: Box Product



$$\vec{B}\times\vec{C}\cdot\vec{D} \equiv \begin{vmatrix} B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \\ D_1 & D_2 & D_3 \end{vmatrix} = \begin{vmatrix} B_1 & C_1 & D_1 \\ B_2 & C_2 & D_2 \\ B_3 & C_3 & D_3 \end{vmatrix} = \varepsilon_{ijk} B_i C_j D_k$$

Problem: $[A]: 3\times 3$ matrix $\Rightarrow |A| = \varepsilon_{ijk} A_{1i} A_{2j} A_{3k} = \varepsilon_{ijk} A_{i1} A_{j2} A_{k3}$

$$|A| = \begin{array}{|ccc|l} A_{11} & A_{12} & A_{13} & B_i \\ A_{21} & A_{22} & A_{23} & C_j \\ A_{31} & A_{32} & A_{33} & D_k \end{array} \Rightarrow |A| = \varepsilon_{ijk} B_i C_j D_k = \varepsilon_{ijk} A_{1i} A_{2j} A_{3k}$$

$$\begin{cases} A_{1i} = B_i \\ A_{2j} = C_j \\ A_{3k} = D_k \end{cases}$$

Problem:

$$\varepsilon_{ijk} = \begin{array}{|ccc|l} \delta_{i1} & \delta_{i2} & \delta_{i3} & \delta_{ir} \equiv B_r \\ \delta_{j1} & \delta_{j2} & \delta_{j3} & \\ \delta_{k1} & \delta_{k2} & \delta_{k3} & \end{array} = \varepsilon_{ijk}\, \delta_{ir}\, \delta_{js}\, \delta_{kt} = \varepsilon_{rst}$$

Problem: $\left| [A] . [B] \right| = |A| . |B|$